math n Motion

Teacher Guide

WESTERN RESERVE
PUBLIC MEDIA

http://www.WesternReservePublicMedia.org/mathnmotion

Table of Contents

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Overview

Math nMotion tackles the very difficult concepts of graphing linear equations, inequalities, ratio and proportion and using data to make decisions through the example of racing Formula M cars.

It is keyed to 8th grade mathematics standards. Each section of this multimedia package contains the following components:

- a three- to four-minute video in .m4v format that gives a brief overview of the concept and real-life examples of why we need to know it
- a set of lesson plans that teach the concept
- a professional development video that explains the main focus of the segment and what is included in each lesson
- a formative and summative assessment for each section

Also included are resource sheets on the Formula M car, the TI-84 graphing calculator and on using Excel.

Credits

Project Coordinator
Maria Mastromatteo, Western Reserve Public Media

Teacher Guide

Teacher Design Team
Cathy Page Adler, Ravenna City School District

Michelle Pirc, Andrews Osborne Academy

Becky Prelog, Kent City School District

Scott Trivisonno, Norton City Schools

Content Expert (and race car driver)
Guy Pipitone

Teacher Guide Layout and Design
Paula Kritz, Western Reserve Public Media

Video

Produced by Western Reserve Public Media (WNEO/WEAO, Youngstown/Akron, Ohio)

Executive Producer
Maria Mastromatteo, Western Reserve Public Media

Producer
Duilio Mariola, Western Reserve Public Media

Videographer
Duilio Mariola, Western Reserve Public Media

Video Script
Larry Chance, Chance Productions

Professional Development Script
Cathy Page Adler, Ravenna City School District

Web

Layout and Design
Paula Kritz, Western Reserve Public Media

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Reference Materials

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About Formula Car Racing

Formula racing is a term that refers to various types of open-wheeled, single-seater cars. An open-wheeled car has wheels outside of the car's main body. The name "formula" was adopted by an international car association, Fédération Internationale de l'Automobile, which is commonly known as FIA. All single-seater racing cars after World War II were called formula cars because they followed "single-seat" regulations or formulae.

We will follow Guy Pipitone, owner and driver of a Formula M car, as he tells us about his efforts to win races. Pipitone uses graphing and algebra to help him be more competitive in races. It is an excellent example of real-world math applications.

Pipitone places sensors around the track that are triggered by a computer in his car. These sensors give him information or data about speed, steering wheel position, engine RPM (revolutions per minute), brake force, G-forces, exhaust gas temperature and gas pedal or throttle position. He then uses this data to make changes in his car that will help him win the race.

Formula Car Facts

- The car is open-wheeled
- The car is about three-quarters of the size of cars at Indianapolis and has much less power
- The car has wings on the back and the front that help keep it on the ground
- Races are not generally public events
- Average lap speeds are usually 90-105 mph
- At the starting line, cars are lined up two-by-two
- There are usually 20-25 cars per race
- The fastest qualifier starts in front and can choose the right or left lane
- Tire temperature is critical
- Tires are a big expense, costing $750 per set and needing replaced twice per weekend of racing
- The cars need tires with no tread on dry days and tires with tread on rainy days
- Three sensors on the tire detect the degree and direction of the tire's tilt
- New Formula M cars cost about $40,000 and used ones cost $20,000
- After each race, the top three winners' cars must be examined by officials to determine the oxygen content of the fuel, the weight of the car and other factors
- Cars make a rolling start, with a lead car taking them around the track for one lap
- There are gravel areas and pull-off areas on the track to be used as needed
- G-force on a Formula One car is 5 Gs in a turn and upwards of 200 mph in the straight part of the track
- The racing season starts in March in the southern United States and May in the north
- All cars are mechanically identical
- Car engines are overhauled and assembled by the same person and must have the same horsepower
- Cars use between six and seven gallons of fuel per race and go 50-60 miles per race, which means they get about 10 miles per gallon
- Generally there is a $300 fee to enter a formula race
- Find out more about formula racing online at Sports Car Club of America, **http://scca.com**, or Data Acquisition Company, **http://advantagemotorsports.com**

TI-84 Graphing Calculator

Some Tips

- **Second Quit** takes you to the home screen
- **Second Enter** replays your earlier steps on your home screen
- **Second 0** gives you a catalog of commands

To Clear the List

- Arrow up to list name
- Hit **Clear**
- Hit **Enter**

To Make a Plot

- Enter the data by going to **Stat** or **Edit**
- Enter data and hit **Enter** key with each entry
- To make a graph:
    - Hit **Second**
    - Hit **Stat Plot**
    - Pick #1, #2 or #3
    - Turn it on
    - Select type of plot you want
    - Select list you want (the default puts List 1 as the x-axis)
- Set the **Window**

To See Specific Points

- Hit the **Trace** key to move around on the screen

To Get Statistics From One List

- Go to **Stat**
- Select **Calc**
- Select **1-var stat**
- Enter list name

To Get a Median Fit Line

- Go to **Stat**
- Select **Calc**
- Select **Med-med** or **#3**
- **Y=**
- Select **Vars**
- Arrow down to **Statistics** and **Enter** or Select **#5**
- Select **EQ (Equations)**
- Select **REG EQ (Regular Equation)** and **Enter** or Select #1
- Hit **Graph**

OR

- Go to **Stat**
- Select **Calc**
- Select **Med-med** or **#3**
- Hit **Second** and whatever columns you're going to use
- Put a comma between entries and after the second entry
- Hit **Vars**
- Select **y-vars** and **Enter**
- Select whichever function you want to use (e.g. Y_1)
- Select **Graph** (make sure you have the screen set correctly)
- Use **Zoom** and select **9** if necessary

To Input a Formula

- Arrow up to the list name and write the formula
- Example List1 + List2

To Generate Random Numbers

- Select **Math**
- Select **PRB**
- Select **RandInt**(
- Insert starting number, ending number and how many in each series, for example:

  RandInt(1,100,5)

To Seed New Calculators for Random Numbers

- Pick a four-digit number
- Select **STO**
- Select **Math**
- Select **PRB**
- Select **Rand**
- Select **Enter**

To Name a List

- Go to **Stat**
- Select **Edit**
- Arrow to the top of the list
- Right arrow to the right to **List 7**
- Make sure the cursor box has an A showing and type in name of list
- If no letter A is showing, go to **Alpha** and type in name of list

To Get a List Back That You Have Lost

- Hit **Stat**
- Select **5—Set Up Editor**

To Use a Named List in a Formula

- With your cursor at any list name, go to **Second List**
- Select the name you want to use

To Use a Name You Have Given a List in a Plot

- Select **Second**
- Select **Stat Plot**
- Select **#1, #2** or **#3**
- Where it says **xlist** or **ylist**, select **Second Stat**
- Arrow down to the list you named
- Select **Enter**
- Select **Graph**

To Delete a List Name You Have Entered

- Go to **Second** and then **Mem**
- Go to **Delete**
- Select what you want to delete by hitting **Enter**
- Be very careful with this; you could erase everything in the memory

To Trace

- Use the left or right arrows (this goes in the order of data entry)
- Select the down arrow to go between two graphs on the screen

To Link

- To send:
    - Choose **Second Link**
    - Select what you want to sent by hitting **Enter**
    - Select **Transmit**
    - Do not press **Enter** until your partner presses receive

- To receive:
    - **Second Link**
    - Select **Receive**
    - Hit **Enter** BEFORE the sender does
    - Your calculator should say **Waiting**
    - After the sender hits **Enter**, select **Overwrite**

If You Get an Error Message

- Check what you have turned on by going to y= and seeing what is highlighted
- Check your window
- Check to see if you have the same number of items in the lists you are using if you are making a scatter plot

APPS Button

- Go to the **APPS button** and select **Probability Simulation (#6)**
- Press any key and you get a menu that includes
    1. Toss Coins
    2. Roll Dice
    3. Pick Marbles
    4. Spin Spinner
    5. Draw Cards and Random Numbers
- Once you have selected a game, the line of blue buttons directly under the screen become active
- If you see the word **Set** in any game, this lets you set the preferences

Excel XP/2003

Naming the Cells

- Use letter and number

Cursors

- Fat Plus Sign — takes you to the cell you want (click and drag to get multiple cells)
- Skinny Plus Sign — allows you to copy (called autofill)
- Arrow — moves the cell(s) to a different location
- Line with two arrows (must be on the line between letters and numbers) — increases the cell size

Alignment

- Numbers to the right.
- Letters to the left.

Highlighting

- Click on a number on the left to highlight an entire row.
- Click on a letter on the top to highlight an entire column.
- Click in the corner box to highlight the entire worksheet.

To Insert Rows, Columns or Sheets

- Go to **Insert**
- Select row, column or worksheet.

To Center

- Highlight item.
- Select to center the entry.

To Undo

- Select the **Undo** button. Continue to press it until you get to the place you want to be.

Selecting Noncontiguous Cells

- Select one, hold the **Control** key down and select another. Both will be highlighted.

Formulas

- Always start with an equal sign.
- Σ means summation. The default is to add vertically. Be careful to highlight anything that you want to add horizontally.
- *ƒ* **x** is the button that gives you the menu of formulas that you might want to use. It is directly in front of the formula bar.
- **$** changes entries to dollars.
- **%** changes entries to percents.
- increases or decreases the number of decimal places.
- moves blocks of text to the left or to the right.
- When inputting your own formula:
  - Start with =.
  - Use *ƒ* **x** to get the name of what you want to use OR just click on the cell to input the cell location.
  - Put cell name where you want the computation to start — then a colon — and then the cell name where you want the computation to end (e.g., =SUM(A1: A7) This will add all of the row from cell A1 to cell A7).
- To keep a number in the same place for a formula (like when computing a percent), you need to use an **Absolute Reference**. In that case, put a **$** before both the letter and the number of the cell you wish to keep. (e.g., A5**/A8** — this will divide the number in cell A5 by A8. It will also divide other numbers by A8.)

Spell Check

- Click the button that looks like this  to check your spelling.

Formatting

- Select **Format**, followed by one of the following:
  - **Cells** lets you change the format of individual cells or rows or columns of cells.
  - Select **Rows**, **Columns** or **Sheet** and then select what you would like to do.
  - Select **AutoFormat** to change the entire worksheet to the style you would like.

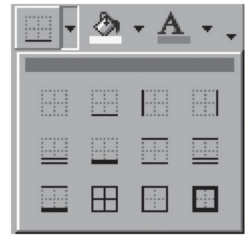
Printing

- If you only want one section of the spreadsheet to print, highlight the section you want printed. Go to **File**, **Print Area** and then **Select Print Area**.
- If you want to center your spreadsheet when you print it, go to **File** and then **Page Setup**. Select **Margins**. At the bottom of the sheet, select **Vertically** and **Horizontally** and then click **OK**.
- If you want your sheet to go on the horizontal layout, select **File** and then **Page Setup**. Then select **Page** and **Landscape**.
- If you want the gridlines to show, go to **Page Setup**. Select **Sheet** and then **Gridlines**.

To Insert a Picture

Go to **Insert**. Select **Picture**. Select **ClipArt** or **From File** depending upon where the picture you want is located. Select the picture you want and click on it. Then select the top button. Close the **ClipArt** page.

To Put in a Border



- Go to the button that looks like this. Click the arrow at the right and then select the border you want to use.

Making a Chart

- Select the Chart icon . Follow the steps of the wizard.

Changing Your Chart

- Select the part of your chart that you would like to alter and right click on it. Select **Format Chart Area** or **Format**. You can change many things. Select **Fill Effects** for many variations.
- If the legend says **Series** and you want to change it to reflect what is on your graph, select **Chart**, then **Source Data** and then **Series.**

Ratio and Proportion

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Chapter Synopsis

Understanding ratio, proportion and percent is a critical skill that is important when solving real-life problems. It is also a stepping stone to understanding more complicated mathematical concepts.

Three lesson plans are provided in this chapter. The first offers an introductory lesson about ratio and proportion. In the second plan, students use given data to compare families' gasoline-powered vehicles to hybrid models. After calculating each vehicle's unit rate of pounds of carbon expelled per gallon of gasoline, they find out which family leaves the greatest carbon footprint.

In the third plan, students use Google Earth to plan a two-week trip itinerary, taking in events at racetracks throughout Ohio. The challenge lies in developing the route that is the most efficient, given time and mileage constraints, to seven different racetrack locations. Students use the Google Earth measuring tool to calculate actual distances in miles and then use proportions to map the trip on an outline map of Ohio. This group of lessons can be used in the math classroom or as a cross-curricular unit with technology and social studies. The formative assessment can also be used as an introduction to learning about Google Earth and its features.

Standards Addressed

Grade 8, Mathematics — Number, Number Sense and Operations

08-10 Benchmark G. Estimate, compute and solve problems involving real numbers, including ratio, proportion and percent, and explain solutions.

Y2003.CMA.S01.G08-10.BG.L08.I05 / Computation and Estimation

05. Determine when an estimate is sufficient and when an exact answer is needed in problem situations, and evaluate estimates in relation to actual answers; e.g., very close, less than, greater than.

Y2003.CMA.S01.G08-10.BG.L08.I06 / Computation and Estimation

06. Estimate, compute and solve problems involving rational numbers, including ratio, proportion and percent, and judge the reasonableness of solutions.

Name____

Formative Assessment: Solving Word Problems With Proportions

For each problem, set up a proportion, being sure to include labels with units. Show all work, including operations. Write a sentence supporting your answer.

1. Farmer Fred's Farm Market offers three pounds of bananas for $1.20. Super Duper Groceries advertises two pounds of bananas for $.98. Which store offers the lower price per pound for bananas?

2. It is 600 miles from Happy Valley to Stormy Bluffs. Monica drove from Happy Valley to Stormy Bluffs, averaging 50 mph. How long was her travel time?

3. Duane read 40 pages of a book in 50 minutes. How many pages should he be able to read in 80 minutes?

4. A collector's model train is scaled so that one inch on the model equals 7½ feet on the actual train. If the model is 2.25 inch high, how high is the actual train? Round to the nearest hundredth.

5. Angela and Michael are trying to determine the distance between two particular cities by using a map. The map key indicates that 4.5 cm is equivalent to 75 km. If the cities are 12.7 cm apart on the map, what is the actual distance between the cities?

Answers for Formative Assessment: Solving Word Problems With Proportions

For each problem, set up a proportion, being sure to include labels with units. Show all work, including operations. Write a sentence supporting your answer.

1. Farmer Fred's Farm Market offers three pounds of bananas for $1.20. Super Duper Groceries advertises two pounds of bananas for $.98. Which store offers the lower price per pound for bananas?

   $ 1.20 / 3 lbs = 1 lb / c dollars

   $ .98 / 2 lbs = 1 lb / d dollars

   c = $.40 / lb.; d = $.49 / lb

   Farmer Fred's Farm Market offers the lower price.

2. It is 600 miles from Happy Valley to Stormy Bluffs. Monica drove from Happy Valley to Stormy Bluffs, averaging 50 mph. How long was her travel time?

   60 miles / x hours = 50 miles / 1 hour

   x = 12 hours

   It takes 12 hours for the trip.

3. Duane read 40 pages of a book in 50 minutes. How many pages should he be able to read in 80 minutes?

   40 pages / 50 min = x pages / 80 min

   x = 64 pages

   Duane should be able to read 64 pages in 80 minutes.

4. A collector's model train is scaled so that one inch on the model equals 7½ feet on the actual train. If the model is 2.25 inch high, how high is the actual train? Round to the nearest hundredth.

   1 in / 7½ ft = 2.25 in / x ft = 16.875 ft, or 16.88 ft

   The actual height of the train is 16.88 ft.

5. Angela and Michael are trying to determine the distance between two particular cities by using a map. The map key indicates that 4.5 cm is equivalent to 75 km. If the cities are 12.7 cm apart on the map, what is the actual distance between the cities?

   4.5 cm / 75 km = 12.7 cm / x km

   x = 212 km

   The actual distance between the two cities is 212 km.

Ratio, Proportion and Percent

Ratio is the comparison of two numbers by division. If a team won three games and lost two games, the ratio of wins to losses could be written in three ways (usually written in lowest terms):

$\frac{3\text{ wins}}{2\text{ losses}}$ $\frac{3}{2}$ 3:2 (read "three to two")

Proportion is a statement of equality for two ratios. For example:

$\frac{3}{6} = \frac{1}{2}$ This could also be written 3:6::1:2, which means three is to six as one is to two.

These very important concepts can help you to solve problems. For example, you spent $25 in two months on your favorite game. If you spend at the same rate, how much can you expect to spend in a year?

$\frac{\text{Months}}{\text{Amount spent}}$ $\frac{2\text{ months}}{\$25} = \frac{12\text{ months (in a year)}}{x\text{ (this is what you want to find)}}$

$2x = 12 \times 25$ or $\frac{\cancel{2}x}{\cancel{2}} = \frac{300}{2}$

$x = \$150$ for the year

A **percent** is the ratio of a number to 100. For example, 8 percent means eight out of 100.

To find a percent of a number, you divide the part by the whole. For example, on your school's bowling team, two-fifths of the team is girls. What percent of the team are girls? To solve, divide two by 5. You will need to add a decimal point and zeros.

2/5 = .4 = 40%

If you move the decimal point two place (past the hundredth place), you will see that 40 percent of the team is girls.

The team has 15 members. How many are girls? This could be solved in two ways.

a. Multiply 15 times 40 percent. Change 40 percent to the decimal .40 by moving the decimal point two places. Then multiply 15 by .4 to get the answer of six girls.

b. Use a proportion to solve the problem.

$\frac{\text{Number of girls}}{\text{Total on the team}}$ $\frac{x\text{ girls}}{15} = \frac{40\text{ (percent of girls)}}{100\text{ (percent on the team)}}$ $x = 6$ girls

Using Ratio and Proportion

Overview

Car racing involves a lot of math. Ratio and proportion become important when drivers and pit crews try to get answers they need to win the race. Students will use ratio and proportion to solve these problems.

Standards Addressed

Grade 8, Mathematics — Number, Number Sense and Operations

08-10 Benchmark G. Estimate, compute and solve problems involving real numbers, including ratio, proportion and percent, and explain solutions.

Y2003.CMA.S01.G08-10.BG.L08.I05 / Computation and Estimation

05. Determine when an estimate is sufficient and when an exact answer is needed in problem situations, and evaluate estimates in relation to actual answers; e.g., very close, less than, greater than.

Y2003.CMA.S01.G08-10.BG.L08.I06 / Computation and Estimation

06. Estimate, compute and solve problems involving rational numbers, including ratio, proportion and percent, and judge the reasonableness of solutions.

Materials

- Calculators

Procedure

1. Ask the students if they have ever gone to a car race. Talk about the different kinds of races, including stock car, NASCAR and formula. Tell them that the type of racing shown in the **Math nMotion** video is Formula M. The "M" stands for Mazda, as these cars have Mazda engines.

2. Students should have some concept of ratio and proportion by eighth grade, but it never hurts to review. Ask some basic questions:

    a. What is the ratio of boys to girls in this class? Girls to boys?

    a. If there are 300 boys in this school and the ratio is the same as this class, how many boys are in the class? How many girls?

    a. How will this proportion look when it is set up?

    a. Is there only one way to set up the proportion?

2. Review using the correct formula for proportion. Review the terms **extremes** (outermost terms) and **means** (inner terms), and different ways of writing proportions:

   a. $\frac{\text{extreme}}{\text{mean}} = \frac{\text{mean}}{\text{extreme}}$

   b. extreme : mean :: mean : extreme

3. Distribute the Using Proportion to Solve Problems handout and do the first example with the students. Show alternate ways the problem could be solved.

Answers to Using Proportion to Solve Problems

1. $\frac{\text{gallons}}{\text{miles}} \quad \frac{1}{6.5} = \frac{x}{55}$

   $\frac{\cancel{6.5}x}{\cancel{6.5}} = \frac{55}{6.5}$

   x = 8.5 gallons

2. $\frac{\text{front}}{\text{back}} \quad \frac{9}{12} = \frac{3"}{4"}$

3. $\frac{\text{width}}{\text{diameter}} \quad \frac{9}{21} = \frac{12}{23}$ $\quad 9 \times 23 \neq 12 \times 21$ $\quad 207 \neq 252$

   not a proportion

4. $\frac{\text{model}}{\text{real car}} \quad \frac{2}{20} = \frac{9.4}{x}$

   $\frac{\cancel{2}x}{2} = \frac{188}{2}$

   x = 94-inch wheelbase on the real car

5. $\frac{\text{model}}{\text{real car}} \quad \frac{2}{20} = \frac{x}{58}$

   $\frac{\cancel{20}\,x}{\cancel{20}} = \frac{116}{20}$

   x = 5.8 inches on the model

Evaluation

CATEGORY	4	3	2	1
Mathematical Concepts	Explanation shows complete understanding of the mathematical concepts used to solve the problem(s).	Explanation shows substantial understanding of the mathematical concepts used to solve the problem(s).	Explanation shows some understanding of the mathematical concepts needed to solve the problem(s).	Explanation shows very limited understanding of the underlying concepts needed to solve the problem(s) or is not written.
Shows Work	Work is detailed and clear.	Work is clear.	Work is somewhat difficult to understand, but includes critical components.	Work is difficult to understand and is missing several components or was not included.
Mathematical Errors	90-100 percent of the steps and solutions have no mathematical errors.	Almost all (85-89 percent) of the steps and solutions have no mathematical errors.	Most (75-84 percent) of the steps and solutions have no mathematical errors.	More than 75 percent of the steps and solutions have mathematical errors.

Name(s) ____________________

Using Proportion to Solve Problems

Car racing involves a lot of math. Ratio and proportion become important when drivers and pit crews try to get answers they need to win the race. Use ratio and proportion to solve these problems. Please show all work!

Example: Tires cost $750 per set. You need two sets (one for the time trial and one for the actual race) per weekend of racing and you race for 10 weekends. What's going to be your cost for the summer? Your ratio is $750/1 race.

$$\frac{\text{Cost}}{\text{Race}} \quad \frac{750}{1} = \frac{x}{20} \text{ (two per week for 10 weeks)}$$

$1x = \$15{,}000$ (just for tires!)

1. A car uses one gallon of fuel to go 6.5 miles. The track is 55 miles long. How much fuel will the driver need?

2. Each front tire is 9 inches wide and each back tire is 12 inches wide. What is the ratio of tires, front to back?

3. The ratio of tire width to tire diameter of the front tires is 9 inches to 21 inches. The ratio for the back tires is 12 inches to 23 inches. Can you set up a proportion with using tire width and diameter? Why or why not?

4. You find a model of a Formula M car. The scale shows that 2 inches on the model equals 20 inches on the car. The wheelbase of the model measures 9.4 inches. What is the actual size of the wheelbase?

5. The distance between the center point of the back tire on the left to the center point of the back tire on the right is 58 inches. (This is called the track rear.) How many inches will that be on the model? Remember, 2 inches on the model equals 20 inches on the car.

A Trail of Carbon Footprints

Overview

Students will use given data to compare gasoline-powered vehicles to hybrid models. After calculating family vehicles' unit rates of pounds of carbon expelled per gallon of gasoline, the students will find out which family leaves the greatest carbon footprint.

Standards Addressed

Grade 8, Mathematics — Number, Number Sense and Operations

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Y2003.CMA.S01.G08-10.BG.L08.I05 / Computation and Estimation

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Y2003.CMA.S01.G08-10.BG.L08.I06 / Computation and Estimation

06. Estimate, compute and solve problems involving rational numbers, including ratio, proportion and percent, and judge the reasonableness of solutions.

Procedure, Day One

1. Ask students if they know what kind of gas mileage their parents get and what kind of cars they drive. Does anyone drive a hybrid vehicle? What is a hybrid? What is a carbon footprint? What does "going green" mean?

2. Pass out the Automobile Carbon Footprint table and discuss the types of models. Which of those listed are compact, midsize, SUVs, etc.? What is the relationship between car size and gas mileage?

3. Because this is a time-consuming and rote activity, the answer key could be distributed in place of the worksheet. An explanation should be given.

4. Introduce unit rate through the context of gas mileage in miles per gallon. List several other unit rates as examples. Why is it easier to express gas mileage as a fractional ratio rather than miles per gallon when performing mathematical calculations?

5. For each model of vehicle, have the students calculate the pounds of carbon per gallon of gas, being sure to label units. Students should perform operations on labels as they do on the ratios. Record results in the table on the recording sheet.

6. Check papers with the class to make sure that they are correct before going on to Day Two exercises.

Answers to Automobile Carbon Footprint

Auto Type and Make (2009)	miles/ gal	miles/ year	mi./year / mi/gal	CO_2 lbs / 100 mi	CO_2 times # miles/yr	CO_2 / yr / # gal/yr
Toyota Highlander Hybrid	26	12,000	12,000 / 26 = 451.5 gal/yr	.75	.75 x 12000 = 9,000 lbs/year	9,000 lbs CO_2 / 451.5 gal 20 lbs/gal
Toyota Highlander	20	12,000	*600 gal*	.98	*11,760 lbs/yr*	*19.6*
Honda Civic Hybrid	42	12,000	*286 gal*	.47	*5,640 lbs/yr*	*19.7*
Honda Civic	30	12,000	*400 gal*	.65	*7,800 lbs/yr*	*19.5*
Mazda Tribute Hybrid	28	12,000	*428 gal*	.70	*8,400 lbs/yr*	*19.6*
Mazda Tribute	21	12,000	*571 gal*	.93	*11,160 lbs/yr*	*19.5*
Saturn Aura Hybrid	30	12,000	*400 gal*	.65	*7,800 lbs/yr*	*19.5*
Saturn Aura	25	12,000	*480 gal*	.82	*9,840 lbs/yr*	*20.5*
Saturn Vue	21	12,000	*571 gal*	.93	*11,160 lbs/yr*	*19.5*
Saturn Vue Hybrid	28	12,000	*428 gal*	.70	*8,400 lbs/yr*	*19.6*
Toyota Camry Hybrid	33	12,000	*363 gal*	.59	*7,080 lbs/yr*	*19.5*
Cadillac Escalade	15	12,000	*800 gal*	1.3	*15,600 lbs/yr*	*19.5*

Source: terrapass.com

Procedure, Day Two

1. Divide the class into groups and give each group a set of Driving Habits handouts.

2. Assign one family per group and instruct the groups to use the miles per gallon unit rates on the Automobile Carbon Footprint Table and the given information about their family to find out how much fuel the family uses.

3. As an extension or challenge activity, have students use the information from the table to find the amount of carbon emissions in pounds per year for one family.

4. After students have determined as a class which family is the greatest environmental offender, have them develop a plan to help the family modify its vehicle and lifestyle choices. They should support their recommendations with mathematical evidence.

Answers to Driving Habits

Guzzler Family

Driver	Destination	Cadillac Escalade Miles / Trip	Saturn Vue Miles / Trip	Number of Miles Dad	Number of Miles Mom
Father	Work	.7 (5 days)		3.5 miles	
Father	Sonny – School	1.5 (5 days)		7.5	
Mother	Work		22 (5 days)		110
Mother	Sissy – School		1.5 (5 days)		7.5
Father	Bowling	3		3	
Mother	Book Club		.5		.5
Mother	Couples Night Out		5		5
Father	Sonny – Practice	1.5 (3 days)		4.5	
Father	Sonny – Running Club	10		10	
Mother	Sissy – Chem. Club		1.5		1.5
Mother	Sissy – Volleyball		1.5 (5 days)		7.5
Mother	Sissy – Job		3 mi. (2 days)		6
			Total Miles	**28.5**	**138**

Amount of CO_2 used by Guzzlers:

Escalade = 28.5 miles/15 mpg = about 2 gallons x19.5 = 39 lbs of carbon

Vue = 138 miles/21 mpg = about 6.6 gallons x 19.5 = 129 lbs

39 + 129 = 168 lbs of carbon for a week

What the family could do to generate less carbon: *Answers will vary. The Guzzlers could get rid of the Escalade because it gets so few miles per gallon. The mom could get a job closer to home and not use so much gas driving to work. The children could ride the bus to school.*

Middleton Family

Driver	Destination	Toyota Camry Hybrid	Honda Civic	Number of Miles Dad	Number of Miles Mom
Father	Work	100		100	
Mother	Joe – Kindergarten	1.5 (5 days)			7.5
Mother	Work		3/wk. at 13 mi.		39
Mother	Susie – School		1.5		1.5
Father	Golfing	2/wk. at 2.5 mi. each	5	5	
Mother	Karate		1.5		1.5
Mother	Volunteer at Food Pantry		5		5
Mother	Joe – T-ball Practice		2/wk. at 3 mi. each		6
Mother	Susie – Library Hour		1		1
Mother	Susie – Swim Lesson		1.5 (3 days)		7.5
Mother	Joe – Science Club		2.3		2.3
Mother	Susie – Ballet Practice		4		4
			Total Miles	**105**	**75.3**

Amount of CO_2 used by Middletons:

Toyota Hybrid = 105 miles/33 mpg = about 3.2 gallons x19.5 = 62.4 lbs of carbon

Honda Civic = 75.3 miles/30 mpg = about 2.5 gallons x 19.5 = 48.75 lbs

62.4 + 48.8 = 111.2 lbs of carbon for a week

What the family could do to generate less carbon: *Answers will vary. The dad could get a job closer to home to use less gas driving to work. The children could ride the bus to school.*

Green Family

Driver	Destination	Honda Civic Hybrid Miles / Trip	Number of Miles
Mother	Work	30 (5 days)	150
Mother	Sissy – School	3 (5 days)	15
Father	Hiking Club	5	5
Father	Workout at Gym	2 (4 days)	8
Mother	Book Club	1.5	1.5
Mother	Tween Camping Assoc.	5	5
Mother	Tween Boy Scouts	1	1
Mother	Teena Envir. Club	3 (2 days)	6
Mother	Teena Book Club	4	4
		Total Miles	**195.5**

Amount of CO_2 used by Greens: *Honda Civic Hybrid = 195 miles/42 mpg = about 4.6 gallons x 19.7 = 91.5 lbs of carbon*

What the family could do to generate less carbon: *Answers will vary. They could get a smaller car. The mom could get a job closer to home to use less gas driving to work.*

Evaluation

Rubric for Driving Habits

CATEGORY	4	3	2	1
Mathematical Errors	Ninety to 100 percent of the steps and solutions have no mathematical errors.	Almost all (85-89 percent) of the steps and solutions have no mathematical errors.	Most (75-84 percent) of the steps and solutions have no mathematical errors.	More than 25 percent of the steps and solutions have mathematical errors.
Strategy/ Procedures	Typically, uses an efficient and effective strategy to solve the problem(s).	Typically, uses an effective strategy to solve the problem(s).	Sometimes uses an effective strategy to solve problems, but does not do it consistently.	Rarely uses an effective strategy to solve problems.
Completion	All problems are completed.	All but one of the problems are completed.	All but two of the problems are completed.	Several of the problems are not completed.
Explanation	Explanation is detailed and clear.	Explanation is clear.	Explanation is a little difficult to understand, but includes critical components.	Explanation is difficult to understand and is missing several components or is not included.

Names ______________________________

Automobile Carbon Footprint

Your goal is to find the amount of carbon dioxide (CO_2) that is used per mile of gasoline for each type of car.

1. Find the number of gallons used in a year.
2. Find the amount of carbon dioxide that is used per mile.
3. Multiply the amount of CO_2 that is used for one mile times the number of miles per year.
4. Divided the amount of carbon dioxide per year by the number of gallons per year.

What does the answer in the last column represent?

Auto Type and Make (2009)	**miles/ gal**	**miles/ year**	**mi./year** / **mi/gal**	**CO_2 lbs** / **100 mi**	**CO_2 times # miles/yr**	**CO_2 / yr** / **# gal/yr**
Toyota Highlander Hybrid	26	12,000	12,000 / 26 = 451.5 gal/yr	.75	.75 x 12000 = 9,000 lbs/year	9,000 lbs CO_2 / 451.5 gal 20 lbs/gal
Toyota Highlander	20	12,000		.98		
Honda Civic Hybrid	42	12,000		.47		
Honda Civic	30	12,000		.65		
Mazda Tribute Hybrid	28	12,000		.70		
Mazda Tribute	21	12,000		.93		
Saturn Aura Hybrid	30	12,000		.65		
Saturn Aura	25	12,000		.82		
Saturn Vue	21	12,000		.93		
Saturn Vue Hybrid	28	12,000		.70		
Toyota Camry Hybrid	33	12,000		.59		
Cadillac Escalade	15	12,000		1.3		

Source: terrapass.com

Names ______________________________

Driving Habits: The Guzzlers

In this activity, you will meet three families: the Guzzlers, the Greens and the Middletons. All of these families lead very active lifestyles and are always looking for ways to trim their budgets. They think that they could save some money by modifying their driving habits.

Study each family's weekly routine and calculate how much is spent on fuel for their cars in traveling to and from work, errands and activities. You should calculate how many gallons of fuel each family uses in one week. You will find the gas mileage amounts on the Automobile Carbon Footprint Table you worked on previously. What recommendations can you make to each family on how to cut driving expenses? Should they purchase a different car? Is their car a good choice already? Should they move? Be sure to support your suggestions with mathematical evidence.

A typical week in the life of the Guzzler Family

Father G. works in the suburbs and drives a Cadillac Escalade so that he can transport Sonny G. and Sissy G. to their activities. Both children are in high school. Mother G. works in the city and drives a Saturn Vue. Below is the Guzzler driving schedule for the week.

Driver	Destination	Cadillac Escalade Miles / Trip	Saturn Vue Miles / Trip	Number of Miles Dad	Number of Miles Mom
Father	Work	.7 (5 days)			
Father	Sonny – School	1.5 (5 days)			
Mother	Work		22 (5 days)		
Mother	Sissy – School		1.5 (5 days)		
Father	Bowling	3			
Mother	Book Club		.5		
Mother	Couples Night Out		5		
Father	Sonny – Practice	1.5 (3 days)			
Father	Sonny – Running Club	10			
Mother	Sissy – Chem. Club		1.5		
Mother	Sissy – Volleyball		1.5 (5 days)		
Mother	Sissy – Job		3 mi. (2 days)		
			Total Miles		

Calculate the amount of CO_2 that the Guzzlers used. Show your work!

Names ______________________________

Driving Habits: The Middletons

In this activity, you will meet three families: the Guzzlers, the Greens and the Middletons. All of these families lead very active lifestyles and are always looking for ways to trim their budgets. They think that they could save some money by modifying their driving habits.

Study each family's weekly routine and calculate how much is spent on fuel for their cars in traveling to and from work, errands and activities. You should calculate how many gallons of fuel each family uses in one week. You will find the gas mileage amounts on the Automobile Carbon Footprint Table you worked on previously. What recommendations can you make to each family on how to cut driving expenses? Should they purchase a different car? Is their car a good choice already? Should they move? Be sure to support your suggestions with mathematical evidence.

A typical week in the life of the Middleton Family

Father M. works out of their home, but he sometimes has business appointments. Mother M. works part-time at City Hospital on Wednesday, Thursday and Friday nights. She takes the children to their activities when she is not working. The children live close enough to school to walk, but their parents drive them daily.

Driver	**Destination**	**Toyota Camry Hybrid**	**Honda Civic**	**Number of Miles Dad**	**Number of Miles Mom**
Father	Work	100			
Mother	Joe – Kindergarten	1.5 (5 days)			
Mother	Work		3/wk. at 13 mi.		
Mother	Susie – School		1.5		
Father	Golfing	2/wk. at 2.5 mi. each	5		
Mother	Karate		1.5		
Mother	Volunteer at Food Pantry		5		
Mother	Joe – T-ball Practice		2/wk. at 3 mi. each		
Mother	Susie – Library Hour		1		
Mother	Susie – Swim Lesson		1.5 (3 days)		
Mother	Joe – Science Club		2.3		
Mother	Susie – Ballet Practice		4		
			Total Miles		

Calculate the amount of CO_2 that the Middletons used. Show your work!

Names ______________________________

Driving Habits: The Greens

In this activity, you will meet three families: the Guzzlers, the Greens and the Middletons. All of these families lead very active lifestyles and are always looking for ways to trim their budgets. They think that they could save some money by modifying their driving habits.

Study each family's weekly routine and calculate how much is spent on fuel for their cars in traveling to and from work, errands and activities. You should calculate how many gallons of fuel each family uses in one week. You will find the gas mileage amounts on the Automobile Carbon Footprint Table you worked on previously. What recommendations can you make to each family on how to cut driving expenses? Should they purchase a different car? Is their car a good choice already? Should they move? Be sure to support your suggestions with mathematical evidence.

A typical week in the life of the Green Family

Papa Green works .5 mi. away from the family home, so he walks or rides his bike to work. Mama Green drives the only family car, a hybrid Cadillac Escalade, to and from work. Tween Green is 10 and he rides the bus to school, and Teena Green is 15 and her mother drops her off at school each morning on her way to work. If Teena has any activities at school, Mama Green drives from their home to get her.

Driver	**Destination**	**Honda Civic Hybrid Miles / Trip**	**Number of Miles**
Mother	Work	30 (5 days)	
Mother	Sissy – School	3 (5 days)	
Father	Hiking Club	5	
Father	Workout at Gym	2 (4 days)	
Mother	Book Club	1.5	
Mother	Tween Camping Assoc.	5	
Mother	Tween Boy Scouts	1	
Mother	Teena Envir. Club	3 (2 days)	
Mother	Teena Book Club	4	
		Total Miles	

Calculate the amount of CO_2 that the Greens used. Show your work!

On the Road Again!

Overview

Students will use Google Earth to plan a two-week trip itinerary, taking in events at racetracks throughout Ohio. The challenge lies in developing the trip that is the most efficient, given time and mileage constraints, to seven different racetrack locations. This group of lessons can be used in the math classroom or as a cross-curricular unit with technology and social studies. The formative assessment can also be used as an introduction to using Google Earth and its features.

Standards Addressed

Grade 8, Mathematics — Number, Number Sense and Operations

08-10 Benchmark **G.** ***Estimate, compute and solve problems involving real numbers, including ratio, proportion and percent, and explain solutions.***

Y2003.CMA.S01.G08-10.BG.L08.I05 / Computation and Estimation

05. Determine when an estimate is sufficient and when an exact answer is needed in problem situations, and evaluate estimates in relation to actual answers; e.g., very close, less than, greater than.

Y2003.CMA.S01.G08-10.BG.L08.I06 / Computation and Estimation

06. Estimate, compute and solve problems involving rational numbers, including ratio, proportion and percent, and judge the reasonableness of solutions.

Materials

- Computers with Google Earth
- Rulers marked in centimeters and millimeters

Previous Knowledge

Finding unit rate; solving proportions with whole numbers.

Procedure

1. If the handout Formative Assessment: Solving Word Problems With Proportions hasn't been completed, have the students complete it now to determine their mastery of calculating using ratio and proportion with rational numbers in decimal form.
2. Distribute rulers and the student handouts Recording Sheet for Distances, Google Map of Racecourses in Ohio, Ohio: The Buckeye State and Trip Itinerary.
3. Introduce the class to Google Earth and its basic features.

4. Have the students find their school and home using the Fly To option and have them label both with placemarks.

5. Allow the students to investigate using the Zoom and Layers features.

6. On the Layers option, under Primary Database, have the students only check Borders and Labels and Roads. This will prevent the picture from becoming overcrowded and confusing.

7. Have students fly to "Ohio + I-70." Check each student's screen to assure that they are in the correct location. Then have them fly to the cities listed on the Trip Itinerary handout, labeling each with a placemark.

8. Students will use Google Earth to help them mark the cities on their map outline and to determine a scale for their maps. Since the segment of I-70 that stretches from Ohio's western border to Columbus is fairly straight, have students use this distance on Google Earth to calculate a unit rate. They should set the measuring tool to miles and record the result to the nearest mile on the handout.

Answer: $$\frac{6.5 \text{ cm}}{1 \text{ cm}} = \frac{120 \text{ miles}}{x \text{ miles}}$$

$$\frac{\cancel{6.5}x}{\cancel{6.5}} = \frac{120}{6.5}$$

Therefore x = 18.5 miles. The scale would be 1 cm = 18.5 miles. This measure is called the unit rate.

9. Students will then determine the order in which they will visit the sites. They will start at their own home and determine the number of miles between each stop.

10. Students should use proportions solved with cross-product equations to calculate the distances on their map outline and label each city in its appropriate location.

11. Students should then find routes that meet the required constraints on the Trip Itinerary handout, determining the most efficient trip in terms of mileage. This task is most easily accomplished by using Google Earth to check conjectures by using the measurement tool and selecting the path feature. The path feature will find the sum of distances along a path made up of distance segments.

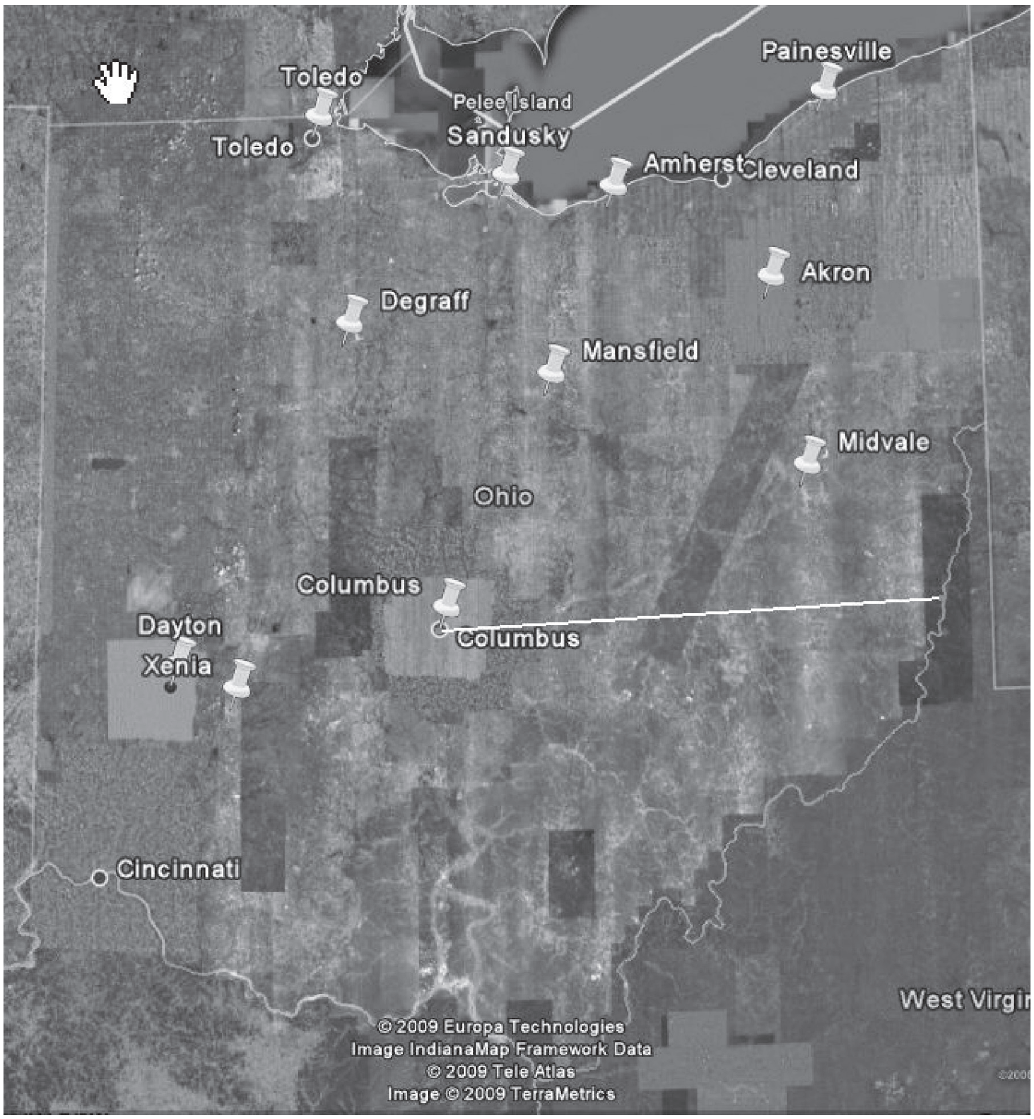
Recording Sheet for Distances

Using the measuring tool on Google Earth, fill in the table below with the distance in miles from city to city. Start with the distance each major city is from Columbus and then fill in the remaining chart with other distances from one major city to another. Continue on another sheet if necessary.

From City	To City	Distance in Miles

Name(s) ______________________________

Google Map of Racecourses in Ohio



Write your stops in order.

Start at ______________

1. ______________

2. ______________

3. ______________

4. ______________

5. ______________

6. ______________

7. ______________

Home

Distance from the eastern Ohio border to Columbus using Rt. 70 is ________miles.

Use proportion to calculate the scale for this map. Show your work.

1 cm = _________miles

Ohio: The Buckeye State

Directions: Draw your route on this sheet. Label your location and use arrows to show the direction you are traveling.

Name(s) ___

Trip Itinerary

The trip includes two weekends. Each day on the itinerary does not have to have a racetrack event scheduled, but remember to visit at least seven tracks listed below. Remember to plan the most efficient trip in terms of miles traveled.

Day	Racetrack Visited
Saturday	
Sunday	
Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	
Sunday	

Racetrack Locations	Days of Scheduled Events
Columbus Motor Speedway, Columbus, Ohio	Thursday through Sunday
Hara Arena, Dayton, Ohio	Tuesday,Wednesday, Saturday
Kil-Kare Speedway, Xenia, Ohio	Saturday, Sunday, Monday
Lorain Speedway, Amherst, Ohio	Friday, Saturday
Mansfield Motorsports Speedway, Mansfield, Ohio	Daily
Midvale Speedway, Midvale, Ohio	Saturday through Tuesday
Painesville Speedway, Painesville, Ohio	Friday, Saturday
Sandusky Speedway, Sandusky, Ohio	Wednesday, Sunday
Shadybowl Speedway, Degraff, Ohio	Friday through Sunday
Toledo Speedway, Toledo, Ohio	Tuesday, Thursday, Saturday

Name ____________________

Summative Assessment

1. Monica read 30 pages of her book in 40 minutes. How many pages should she be able to read in one hour?

2. Franny wanted some new jeans. Her mom told her how many pairs she already had. She found she had six T-shirts for every two pairs of jeans. She had 30 T-shirts. How many pairs of jeans did she have?

3. You can buy six oranges for $2.16. How much will 10 oranges cost?

4. Michael earned four days of vacation for every three months he works. How much vacation will he earn in two years?

5. Heidi is planning to visit Germany. She found out that 1.4 American dollars is equal to .71 euro (the currency in Europe). She wants to carry some money with her so she cashes in $25 in American currency. How many euros should she receive?

6. Miguel was making a miniature painting for a class project. The original was 32 inches tall and 42 inches wide. The miniature will have a height of 4 inches. What will the width be?

7. Hot Wheels cars have a 1:64 ratio to a real car. If my car is 16 feet long, $6\frac{3}{5}$ feet wide, and $5\frac{3}{4}$ feet tall, what are the dimensions of the Hot Wheels version of my car in inches? Give your answers rounded to the nearest tenth.

   length = _____ inches width = _____ inches height = _____ inches

8. You take a 4-inch by 6-inch photograph and blow it up to 8 inches by 10 inches. Are the two pictures similar? Why or why not?

9. Physics tells us that weights of objects on the moon are proportional to their weights on Earth. The quarterback of the Brown Middle School football team weighs 180 pounds on Earth, or 30 pounds on the moon. How much would you weigh on the moon? (You may use any weight for this problem.)

10. A can of spray paint costs $3.99 for three ounces. A different brand costs $5.99 for six ounces. Which can is the better buy? Prove your answer.

Bonus: Back to the beginning! To determine the number of deer in a forest, a forest ranger tags and releases 300 of them. Later, 415 deer are caught, out of which 51 of them are tagged. About how many deer are in the park?

Summative Assessment Answers

Presented here is one possible proportion for each problem. Students need to realize that they may have set their proportion up in a different way and it could still be correct.

1. $\frac{\text{pages read}}{\text{minutes}} \quad \frac{30}{40} = \frac{x}{60} \quad x = 45 \text{ pages}$

2. $\frac{\text{shirts}}{\text{jeans}} \quad \frac{6}{2} = \frac{30}{x} \quad x = 10 \text{ pairs of jeans}$

3. $\frac{\text{oranges}}{\text{cost}} \quad \frac{6}{2.16} = \frac{10}{x} \quad x = \$3.60 \text{ cost}$

4. $\frac{\text{vacation days}}{\text{months}} \quad \frac{4}{3} = \frac{x}{24} \quad x = 32 \text{ days vacation}$

5. $\frac{\text{dollar}}{\text{euro}} \quad \frac{1.4}{.71} = \frac{25}{x} \quad x = 12.7 \text{ euros}$

6. $\frac{\text{length}}{\text{width}} \quad \frac{32}{42} = \frac{4}{x} \quad x = 5.25 \text{ inches}$

7. Length: 16 ft = 192 inches

   $\frac{\text{Hot Wheels}}{\text{car}} \quad \frac{1}{64} = \frac{x}{192} \quad x = 3 \text{ inches long}$

   Width: $6\frac{3}{5}$ = 6.6 ft = 79.2 inches

   $\frac{\text{Hot Wheels}}{\text{car}} \quad \frac{1}{64} = \frac{x}{79.2} \quad x = 1.24 \text{ inches wide}$

   Height: $5\frac{3}{4}$ = 5.75 ft = 69 inches

   $\frac{\text{Hot Wheels}}{\text{car}} \quad \frac{1}{64} = \frac{x}{69} \quad x = 1.1 \text{ inches high}$

8. $\frac{4}{6} \neq \frac{6}{10}$ Therefore, pictures will not be similar

9. Answers will vary

10. $\frac{\text{cost}}{\text{ounces}} \quad \frac{3.99}{3} = \frac{x}{1} \quad x = \$1.33 \text{ per ounce}$

    $\frac{\text{cost}}{\text{ounces}} \quad \frac{5.99}{6} = \frac{x}{1} \quad x = \$.99 \text{ per ounce}$
    – the better buy

Bonus:

$\frac{\text{sample tagged}}{\text{sample total}} \quad \frac{51}{415} = \frac{\text{population tagged}}{\text{population total}} \quad \frac{300}{x}$

x = 2,441 deer

Ratio and Proportion Vocabulary

Percent: Another way of saying hundredth, or divided by 100. It is usually denoted by the symbol %.

Proportion: An equation that states that two ratios are equal.

Ratio: The ratio of one number to another number is the quotient when the first number is divided by the second number (not zero).

Solution of a sentence: Any value of a variable that turns an open sentence into a true statement.

Unit price: The price of one unit of a given item.

Variable: A symbol used to represent one or more numbers.

math in Motion

Graphing Linear Equations

WESTERN RESERVE
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Chapter Synopsis

The coordinate plane is a two-dimensional surface on which we can plot points, lines and curves. It has two scales, called the x-axis and y-axis, at right angles to each other that meet at the point of origin (0, 0). Rather than teaching graphing equations using the coordinate plane in an abstract format, students are given real-life examples of how and why they could create and use a line to give them information that will help them to solve problems.

In the first lesson, students create a ramp and record both time and distance of miniature cars going down the ramp. They do this three times and find the average rate. They then interpolate other times and create a graph of the data.

The students follow a lemonade stand operation in the second lesson to determine what will make the stand successful. They use algebraic equations to help make that determination.

In the third lesson, students create a walking course and record the time and distance walked. They create a graph of their times and use an equation to find the rate.

Formative and summative assessments are available, as well as an overview of graphing linear equations and a vocabulary sheet.

Standards Addressed

Grade 8, Mathematics — Patterns, Functions and Algebra

08-10 Benchmark ***D. Use algebraic representations, such as tables, graphs, expressions, functions and inequalities, to model and solve problem situations.***

Y2003.CMA.S04.G08-10.BD.L08.I07 / Use Algebraic Representations

07. Use symbolic algebra (equations and inequalities), graphs and tables to represent situations and solve problems.

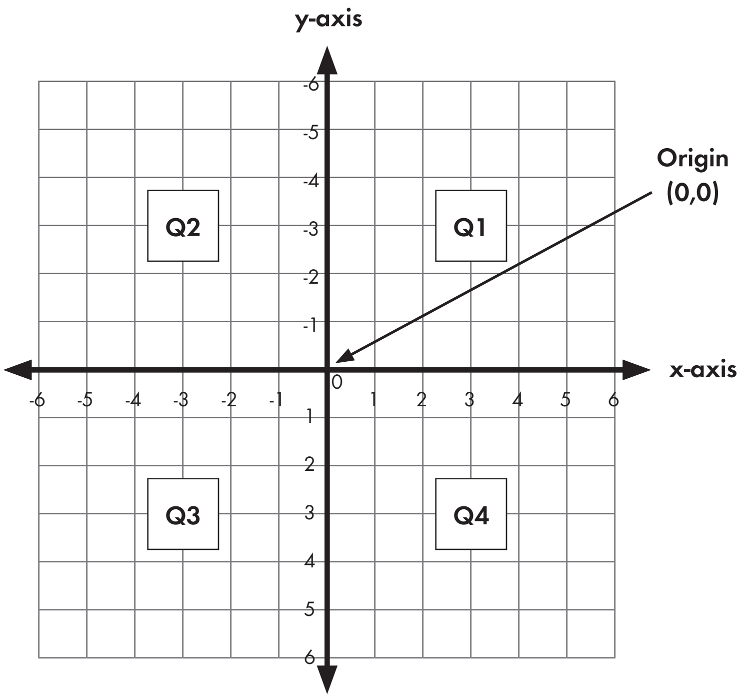
Y2003.CMA.S04.G08-10.BD.L08.I08 / Use Algebraic Representations

08. Write, simplify and evaluate algebraic expressions (including formulas) to generalize situations and solve problems.

Graphing Linear Equations

An equation represents a scale. Instead of keeping the scale balanced with weights, numbers and symbols are used. These numbers are called **constants** because they constantly have the same value. The equation may also be balanced by using a **variable**. A variable is an unknown number represented by any letter in the alphabet (often x). The value of each variable must remain the same in each problem but varies from problem to problem.

Equations can be plotted on a graph using ordered pairs. The graphs that these equations are plotted on are called Cartesian planes or coordinate planes. They look like the picture below.



There are four quadrants. They are numbered counterclockwise. Quadrant 1 starts at the top left.

An ordered pair has the form (x, y). X points move right or left of the origin. Y points move up or down from the origin (0,0).

Quadrant 1 – (+, +)

Quadrant 2 – (-, +)

Quadrant 3 – (-, -)

Quadrant 4 – (+,-)

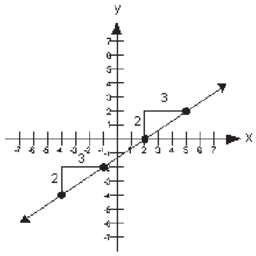
Lines on a coordinate plane are commonly written in the **slope-intercept** form. The **slope** of a line measures the steepness of the line and is generally represented by the variable m. The **intercept** is where the line crosses the y-axis and is generally represented by the variable b.

The slope is thought of as $\frac{\textbf{rise}}{\textbf{run}} = \frac{\textbf{vertical change}}{\textbf{horizontal change}} = \frac{y_2-y_1}{x_2-x_1}$

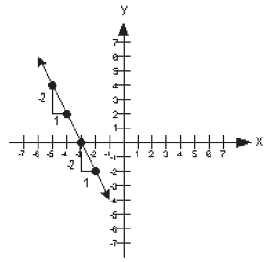
For all real numbers, the slope-intercept form for the equation of a line is y = mx + b.

Slopes can be positive, negative, zero or undefined.

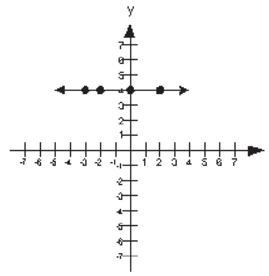
Positive Slope



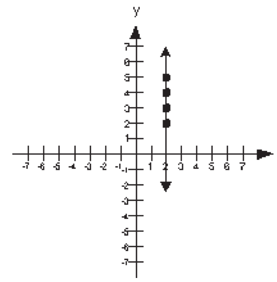
Negative Slope



Zero Slope



Undefined Slope



Slope

Given two points (2, 0) and (3,7), the slope can be found by dividing the difference of the y's by the difference of the x's.

$$\frac{y_2\text{-}y_1}{x_2\text{-}x_1} = \frac{7\text{-}0}{3\text{-}2} = \frac{7}{1}$$

Therefore, the slope of the line is 7.

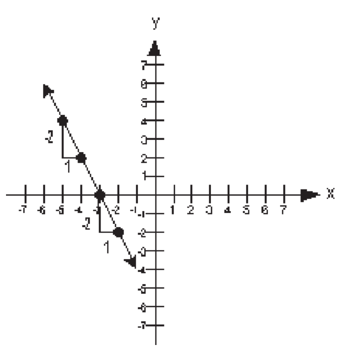
Once you know the slope, you can substitute it in the equation. You also know the value of x and y because of the points in your ordered pair. You can derive the equation of the line.

Using the above slope:	y	=	mx + b
	7	=	7(3) + b
	7	=	21 + b
	b	=	7-21 or -14

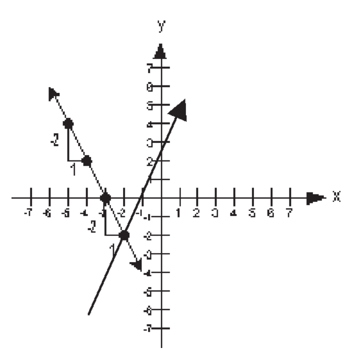
The equation of the line is y = 7x -14.

Plotting a Line on a Coordinate Plane

In the equation y = -2x -5, -2 is the slope and -5 is where it crosses the y-axis or the y-intercept. It would look like this:



Many times a situation could occur where there is more than one relationship between the x and y variables. The two lines could have the same (x, y) values simultaneously. These would be intersecting lines. This point is often significant. The graphs might look like this.



What's My Velocity?

Overview

Students will create a ramp and record both time and distance of toy cars going down the ramp. They will do this three times and find the average rate. They will then interpolate the times and create a graph of the data.

Standards Addressed

Grade 8, Mathematics — Patterns, Functions and Algebra

08-10 Benchmark D. Use algebraic representations, such as tables, graphs, expressions, functions and inequalities, to model and solve problem situations.

Y2003.CMA.S04.G08-10.BD.L08.I07 / Use Algebraic Representations

07. Use symbolic algebra (equations and inequalities), graphs and tables to represent situations and solve problems.

Y2003.CMA.S04.G08-10.BD.L08.I08 / Use Algebraic Representations

08. Write, simplify and evaluate algebraic expressions (including formulas) to generalize situations and solve problems.

Materials

- 15 die-cast toy cars, such as Matchbox or Hot Wheels
- 10-ft piece of cardboard or other hard surface
- 15 stopwatches
- Tape
- Stairway or incline

Procedure

1. You will need an incline for this experiment. You can use stairs in your school or you can set up a box or crate and piece of cardboard so the ramp will cause the toy cars to roll downhill.
2. Students will be mimicking the straightaway speed of an Indy car by rolling toy cars down ramps. You can have ramps at different heights if you would like groups to analyze the difference in rates of different ramps. (Compare the speeds to the severity of the ramps' slopes.)
3. Divide the class into groups. Have them run three trials of their cars and calculate the average time it took for the car to travel the given distance of x feet. Have them fill out the table of all three trials and the average.

4. Once the students have their average time, have them calculate the distance the car would travel over one second. By this time in the students' math careers, they should be familiar with proportions and ratios. Have them set up a proportion of their distance over time equal to x distance over one second:

$$\frac{\text{Distance}}{\text{Time}} = \frac{x \text{ distance}}{1 \text{ second}}$$

   (They might not see this yet, but they are calculating what is called the "unit rate.")

5. This is a good time to check the groups' work to assure that they have recorded their data correctly and have not made any major mistakes that will alter their calculations.

6. Now that each group has the unit rate for its car, have them share their information with other groups so that each car has its own unit rate. Make sure to create a note of which ramp the data paired up with, if you used different ramp heights.

7. Students have their rate and should use this to create a table and graph of the car traveling over x amount of time and y number of feet. The table should start at zero and increase by one in the x-axis (time).

8. Students should notice in the table that it increases a constant amount each time x increases by one. They should also notice that the graph is a straight line with no bumps or curves, increasing by a constant rate.

9. Have the groups use their table and graph to verify the distance the car would travel if it had continued at that speed for 10 seconds, 15 seconds and 20 seconds. (It is called **extrapolation** when a student makes a prediction based on the data given.)

10. To calculate the distances of greater times, have students create an equation to represent the situation of their car traveling down the ramp.

11. Introduce them to the equation D = rt. Students can write the equation as y = rx also.

12. Use the worksheet to summarize the day's lesson and use the questions in the Summative Evaluation to help guide student thinking.

Evaluation

Rubric for Graphs

CATEGORY	4	3	2	1
Title	The title is creative and clearly relates to the problem being graphed. It is printed at the top of the graph.	The title clearly relates to the problem being graphed and is printed at the top of the graph.	A title is present at the top of the graph.	A title is not present.
Labeling of X-axis	The x-axis has a clear, neat label that describes the units used.	The x-axis has a clear label.	The x-axis has a label, but it is unclear.	The x-axis is not labeled.
Labeling of Y-axis	The y-axis has a clear, neat label that describes the units.	The y-axis has a clear label.	The y-axis has a label, but it is unclear.	The y-axis is not labeled.
Accuracy of Plot	All points are plotted correctly and are easy to see. A ruler is used to neatly connect the points or make the bars, if not using a computerized graphing program.	All points are plotted correctly and are easy to see.	All points are plotted correctly.	Points are not plotted correctly or extra points were included.
Units	All units are described (in a key or with labels) and are appropriately sized for the data set.	Most units are described (in a key or with labels) and are appropriately sized for the data set.	All units are described (in a key or with labels) but are not appropriately sized for the data set.	Units are neither described nor appropriately sized for the data set.
Neatness and Attractiveness	Exceptionally well designed, neat and attractive. Colors that go well together are used to make the graph more readable. A ruler and graph paper (or graphing computer program) are used.	Neat and relatively attractive. A ruler and graph paper (or graphing computer program) are used to make the graph more readable.	Lines are neatly drawn but the graph appears quite plain.	Appears messy and produced in a hurry. Lines are visibly crooked.
Concepts	Student has a clear understanding of plots and has answered the questions effectively.	Student has satisfactory understanding of the major concepts but has small misunderstandings.	Student has major misunderstandings of the concepts and cannot complete work on his or her own.	Student does not display understanding of the major concepts or did not complete the assignment.

Name(s) ____________________

Speed!

One type of car racing is Formula M racing. These cars travel on tracks that sometimes curve and wind through cities. They are similar to Formula 1 cars but are not as powerful. Each car must weigh greater than 1,350 pounds and have a Mazda engine. Racing officials regulate these cars carefully. Racetracks are about 50-60 miles long and the car takes about six to seven gallons of fuel. During a race, an average Formula M car travels 100 mph, which is 126 feet per second. The cars can costs from $20,000 to $40,000.

Ramp #: ________ Ramp Height: ________ Ramp Length: ________ Car Name: ____________________

Trial 1		**Trial 2**		**Trial 3**		**Average Rate**
Distance	**Time**	**Distance**	**Time**	**Distance**	**Time**	
*Rate =		*Rate =		*Rate =		

* Be sure to label the rate

1. Write down the distance your car traveled in your measured time.

2. Calculate the unit rate by creating a proportion of the rate you measured to a distance over one second. If your distance was 25 feet and you did it in five seconds, you could use a proportion to find the unit rate.

$$\frac{\text{distance}}{\text{time}} = \frac{x\ \text{distance}}{1\ \text{second}} \quad \text{so} \quad \frac{25}{5} = \frac{x}{1}$$

$$\frac{\cancel{5}\,x}{\cancel{5}} = \frac{25}{5}$$

$$X = \frac{5\ \text{feet}}{1\ \text{second}} \qquad \text{This is the unit rate.}$$

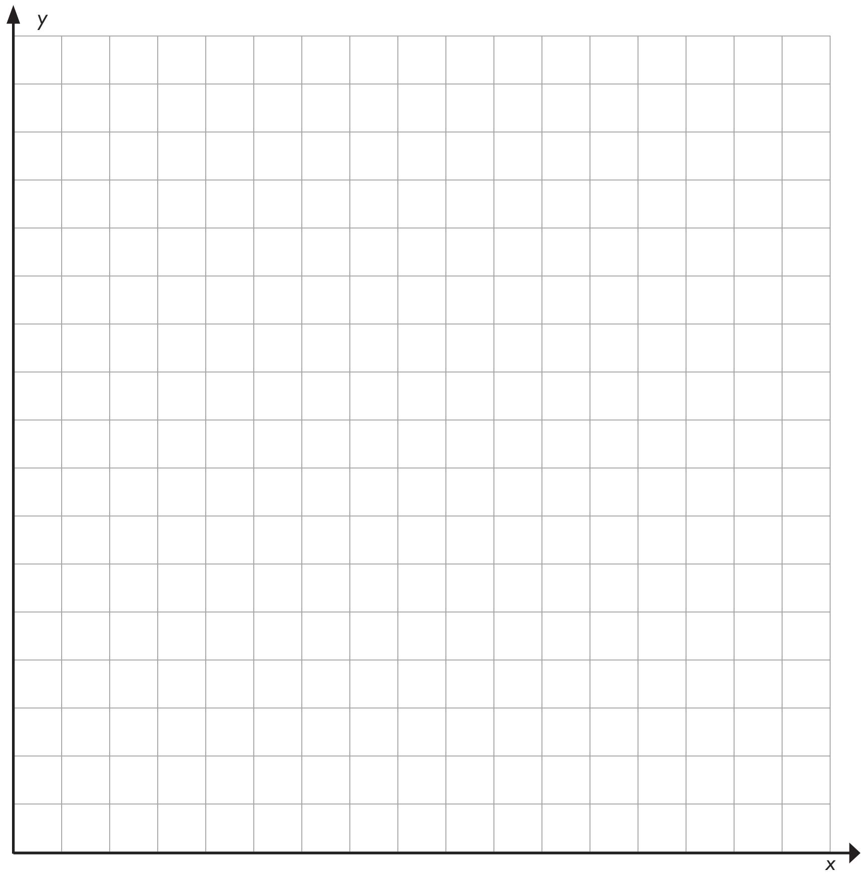
Calculate your unit rate here.

3. Ask your teacher to check your progress at this point.

4. Now you are ready to share your unit rate with the class.

5. Fill in the table below. You are going to use your unit rate to show the distance the car traveled after each number of seconds passing. Use the table to create the graph.

Time (Seconds)	Distance (Feet)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	



6. Use the table and graph to answer the next question. How far would the car travel after 10, 15 and 20 seconds? How does the answer show up in the table? How does the answer show up in the graph?

7. As your seconds increase by one, what do you notice about the distance the car travels? How is this pattern shown in the graph?

8. Use the table and write an equation that would calculate the distance (d) your car traveled after x amount of time. Have your teacher check your work again.

9. Use the above equation to calculate the distance your car would travel after 30, 60 and 120 seconds.

10. If your car traveled 100 feet, how long was your car going down the ramp?

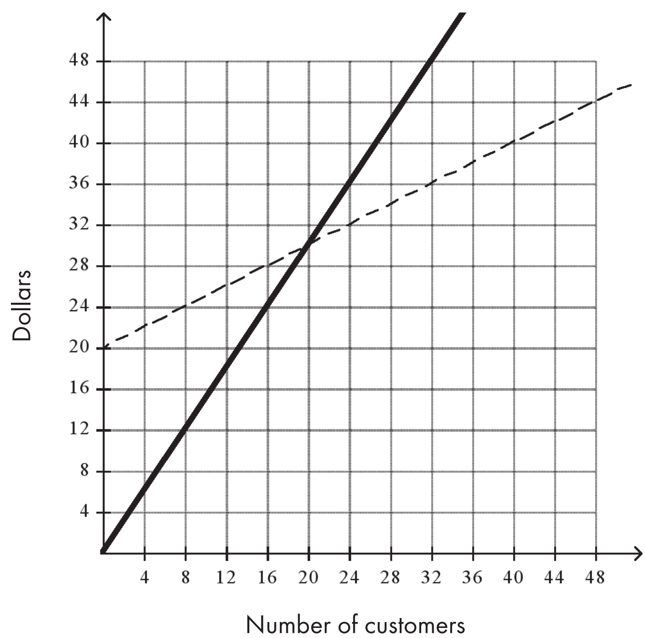
11. What is different in how you solved the question for question seven and question for eight?

12. What could be changed about this experiment to have your car travel further after 30, 60 and 120 seconds? How would this affect your table? How would this affect your graph?

13. If given a table or graph, describe the steps involved when writing a linear equation. Be sure to use the keywords slope and y-intercept in your explanation.

Name____

Formative Evaluation



Scott and Becky had a lemonade stand. This graph shows the story of their business. Write a story to explain the graph.

The dashed line is expense.

The solid line is income.

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Algebraic Representation

Overview

Students will use algebraic equations to determine how to make a lemonade stand profitable.

Standards Addressed

Grade 8, Mathematics Patterns, Functions and Algebra

08-10 Benchmark D. Use algebraic representations, such as tables, graphs, expressions, functions and inequalities, to model and solve problem situations.

Y2003.CMA.S04.G08-10.BD.L08.I07 / Use Algebraic Representations

07. Use symbolic algebra (equations and inequalities), graphs and tables to represent situations and solve problems.

Y2003.CMA.S04.G08-10.BD.L08.I08 / Use Algebraic Representations

08. Write, simplify and evaluate algebraic expressions (including formulas) to generalize situations and solve problems.

Procedure

1. Start by showing the Getting Started graph as an overhead and analyzing the data with the students. Discuss and review linear equations in a graph, table and equation.

2. Now that you have reviewed the parts of linear equations, review why the labels are given to the y-axis and x-axis. Discuss independent vs. dependant variables. You can also discuss continuous vs. non-continuous graphs. (Should the dots be connected?)

3. Introduce the scenario with Sarah and Stephanie and be sure to identify "I can" statements for the lesson:

    a. "I can use a table, graph or equation to determine if it's a linear or non-linear function."

    b. "I can find the slope and y-intercept of a line and use it to write an equation and vice versa."

    c. "I can use a table to make a graph, and use a graph to make a table."

4. This lesson is a very student-centered lesson, which forces the students to answer the questions from the reading and the teacher to work around the room, posing questions when students are lost or need more of a challenge.

5. Find several checkpoints that allow discussion with the partner pairs. Use them as opportunities to check students' work. This assures that students are on task and on target.

6. Summarize the lesson with the students by reviewing it or selecting major questions that you find important. Be sure that all students have correctly written an equation and created the table and graph.

Getting Started Answers

- What information can we extract from that graph?
  We can extract that a person is recording the number of waves they are receiving during the time they mow the lawn

- What is the relationship between the number of waves and time?
  As time passes, the number of waves is increasing at a constant rate.

- What is the slope of this graph? What is the y-intercept of this graph?
  The slope shows that the person is receiving four waves an hour. They started with zero waves.

- Explain how to identify the slope and y-intercept in this graph.
  Calculate the slope by counting the rise over the run from one point to the next. The y-intercept can be identified with the line crosses the y–axis.

Summative Evaluation

1. "Explain the steps when writing a linear equation."

2. "What are the two major parts in a linear equation and how are they identified in a table, graph and equation?"

3. "What was the independent variable in both the income and expense equations?"

4. "Why did both equations depend on the number of customers?"

5. "At what moment were you supposed to identify in the graph?"

6. "What did this moment represent?"

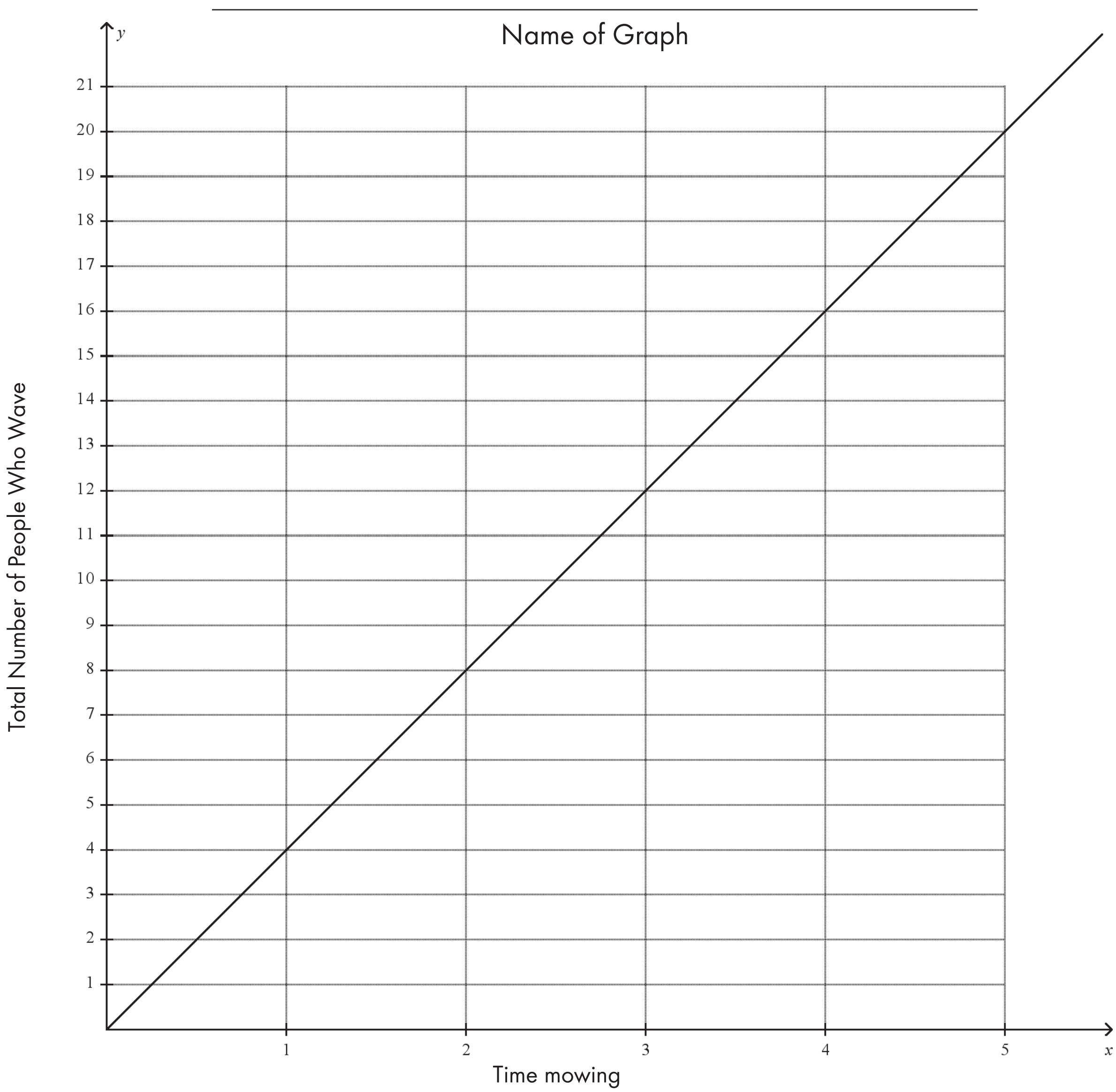
7. "For the two graphs to cross, what must be true of their rates?"

8. "If the rates of two linear lines were the same, what would be significant about that graph?"

9. "For the girls to make a profit, what must they do?"

Name____

Getting Started



- This graph shows the number of people that wave to me as I mow my lawn during a typical summer day.
- What information can we extract from that graph?
- What is the relationship between the number of waves and time?
- What is the slope of this graph? What is the y-intercept of this graph?
- Explain how to identify the slope and y-intercept in this graph.

Name(s) ____________________

Algebraic Representation: Lemonade Stand

Sarah and Stephanie are sisters who started a lemonade stand called Cool Drinks. They decided to split the responsibility and to work equally. Sarah is in charge of buying the supplies to start their stand, while Stephanie is in charge of working the stand and making it operational in its first week. Sarah travels to the store to buy lemonade mix, cups, pitchers, filtered water, a table and chairs. Sarah first collects items that only need to be bought one time, such as pitchers, table and chairs. This cost comes to a grand total of $40. Sarah reports back to Stephanie and informs her of the prices of the other items that must continue to be replenished. These items include:

- One gallon of filtered water: $0.78
- One pack of lemonade mix: $3.72
- One pack of 16 cups: $1.10

This combination will supply enough water, mix and cups to serve 16 customers. Now it is Stephanie's turn to set up the lemonade stand.

1. Below, write how Stephanie may use the above information to make sure that Cool Drinks will be a successful business.

2. Stephanie decides that each cup of lemonade will cost $0.85. What about this price will make Cool Drinks successful or unsuccessful?

3. Write an equation below that represents the total expense (E) the girls will have for buying supplies and selling a cup (x) of lemonade. (Hint: Make sure to calculate the cost of one cup of lemonade.)

4. Write an equation below that represents the total income (I) the girls will have from selling a cup (x) of lemonade to visiting patrons.

5. In the first two days of business the girls have 25 customers. Calculate the total expense to run Cool Drinks and the total income.

Based on the income and expenses the girls had, did they have a successful two days?

6. Make a table and graph for the total *expense* the girls will have for the first 100 customers. On the same table and same graph, add the total *income* the girls will have for the first 100 customers. Use different colors on your graph for income and expense and use a scale of 10 in the table and the graph.

7. Use your graph or table to estimate the number of customers it will take Sarah and Stephanie to have the same amount of income as they do expense. When using the graph, how is the moment shown?

8. At what number of customers will Cool Drinks be a successful lemonade stand?

9. Profit is the amount of money that a business can keep after it pays for its expenses. It can be calculated by taking the income and subtracting the expense. Use the model below to write a profit equation for Cool Drinks. Be sure to simplify the equation Profit = Income – Expense.

10. If Sarah and Stephanie serve 100 customers, what will be their profit?

11. Write a story below that explains Sarah and Stephanie's day selling lemonade and that coincides with the graph.

Algebraic Representation: Lemonade Stand Answers

1. Below, write how Stephanie may use the above information to make sure that Cool Drinks will be successful and worthwhile.

   Stephanie wants to make sure that the rate at which she sells her lemonade is higher than the rate that it costs to make a cup of lemonade.

2. Stephanie decides that each cup of lemonade will cost \$0.85. What about this price will make Cool Drinks successfully or unsuccessful?

   Cool Drinks will be successful because she made the cost of a cup of lemonade \$0.50 more than it is to make a cup of lemonade.

3. Write an equation below that represents the total expense (E) the girls will have for buying supplies and selling a cup (x) of lemonade. (Hint: Make sure to calculate the cost of one cup of lemonade.)

   Expense: $E = .35x + 40$

4. Write an equation below that represents the total income (I) the girls will have from selling a cup (x) of lemonade to visiting patrons.

   Income: $I = .85x$

5. In the first two days of business the girls have 25 customers. Calculate the total expense to run Cool Drinks and the total income. Based on the income and expenses the girls had, did they have a successful two days?

   Income: $I = .85x$ *Expense:* $E = .35x + 40$

   Income: $I = .85(25)$ *Expense:* $E = .35(25) + 40$

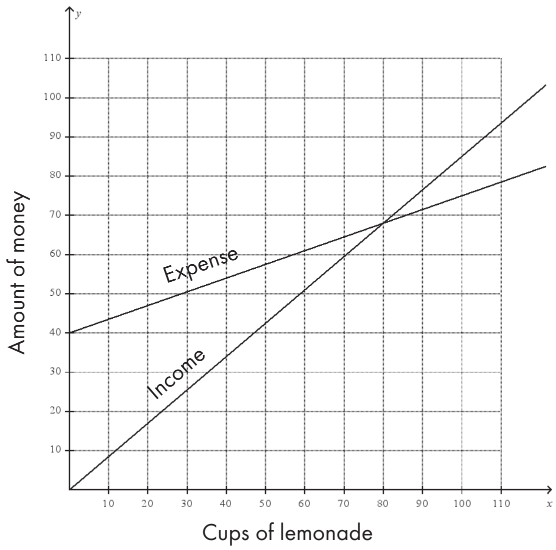
   Income: $I = \$4.25$ *Expense:* $E = \$48.75$

   No it was not a successful two days because the business spent \$44.50 more than they brought in.

6. Make a table and graph for the total *expense* the girls will have for the first 100 customers. On the same table and same graph, add the total *income* the girls will have for the first 100 customers. Use different colors on your graph for income and expense. (use a scale of 10 in the table and the graph)

7. Use your graph or table to estimate the number of customers it will take Sarah and Stephanie to have the same amount of income as they do expense. When using the graph, how is the moment shown?

The point of intersection between the income line and the expense line will represent when the two amounts are equivalent.



Cups	0	10	20	30	40	50	60	70	80	90
Money(I)	0	8.50	17	25.50	34	42.50	51	59.50	68	76.5
Money(E)	40	43.50	47	50.50	54	57.50	61	64.50	68	61.50

8. At what number of customers will Cool Drinks be a successful lemonade stand?

Cool Drinks will be successful when it sells 80 cups of lemonade

9. Profit is the amount of money that a business can keep after it pays for its expenses. It can be calculated by taking the Income and subtracting the expense. Use the model below to write a profit equation for Cool Drinks. Be sure to simplify the equation Profit = Income – Expense.

Profit = $(.85x) – (.35x + 40)$

Profit = $.85x - .35x – 40$

Profit: $P = .5x - 40$

10. If Sarah and Stephanie serve 100 customers, what will be their profit?

Profit: $P = .5x - 40$

Profit: $P = .5(100) - 40$

Profit: $P = \$10.00$

Algebraic Representation: Lesson Assessment

Chelsea and Drew started a small business selling fruits and vegetables to students in the morning. They came up with two equations, one to represent the amount of money they spend to purchase fruits and vegetables (expenses) and one to represent the amount of money they bring in (income). Create a profit equation that will combine and simplify the expense and income.

Expense: $E = .5x + 60$

Income: $I = 2.00x$

Profit Equation:

1. How much money will Chelsea and Drew earn if they sell a total of 20 fruits and vegetables? What does this number mean?

2. How much money will Chelsea and Drew earn if they sell a total of 50 fruits and vegetables? What does this number mean?

3. How many fruits and vegetables will Chelsea and Drew need to sell in order to break even? (This means that their profit would be zero.)

4. Graph the profit equation below and explain how to use this graph to distinguish the difference between losing money and gaining money depending on the number of students.

5. Compare this graph with the one from the lemonade problem. Discuss the similarities and differences between the previous graph and this graph when showing profit.

Algebraic Representation:
Lesson Assessment Answers

Chelsea and Drew started a small business selling fruits and vegetables to students in the morning. They came up with two equations, one to represent the amount of money they spend to purchase fruits and vegetables (expenses) and one to represent the amount of money they bring in (income). Create a profit equation that will combine and simplify the expense and income.

Expense: $E = .5x + 60$

Income: $I = 2.00x$

Profit Equation: $P = 1.5x - 60$

1. How much money will Chelsea and Drew earn if they sell a total of 20 fruits and vegetables? What does this number mean?

   Chelsea and Drew will earn -$30. This means that they will be "in the hole" $30, or that they have not broken even and still have spent more than they earned.

2. How much money will Chelsea and Drew earn if they sell a total of 50 fruits and vegetables? What does this number mean?

   Chelsea and Drew will earn $15. This means they have made $15 more than they have spent on fruits and vegetables.

3. How many fruits and vegetables will Chelsea and Drew need to sell in order to break even? (This means that their profit would be zero.)

   $P = 1.5x - 60$

   $0 = 1.5x - 60$

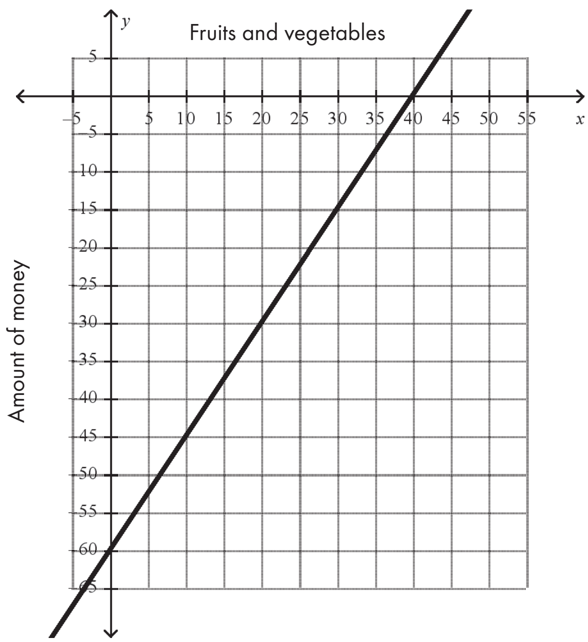
   $60 = 1.5x$

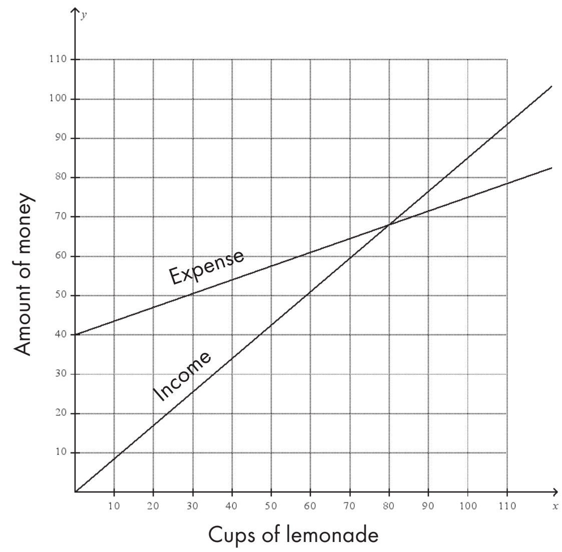
   $40 = x$

   Chelsea and Drew will have to sell 40 fruits and vegetables total to break even.

4. Graph the profit equation below and explain how to use this graph to distinguish the difference between losing money and gaining money, depending on the number of students.

   Compare this graph with the one from the lemonade problem. Discuss the similarities and differences between the previous graph and this graph when showing profit.





The break even point for the lemonade stand problem was when the income and expense lines intersected. Here we see the break even point when the line crosses the x-axis.

Finding Rates

Overview

Students will create a walking course and record the time and the distance walked. They will create a graph of their times and use an equation to find the rate.

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Y2003.CMA.S04.G08-10.BD.L08.I08 / Use Algebraic Representations

08. Write, simplify and evaluate algebraic expressions (including formulas) to generalize situations and solve problems.

Materials

- Tape
- Stopwatches

Procedure

1. Prepare a course beforehand by marking the floor at consistent intervals, such as every foot or every 30 centimeters. You may use standard or metric measurements, depending upon your preference.

2. Divide the class into groups of three. In each group, one student will be the walker, one the timer and the third the measurer.

3. Hand out to each group a copy of the
   - A worksheet titled *How Do I Rate?*
   - A stopwatch

4. Explain the introduction to the students and demonstrate how to walk the course. Emphasize that they should try to walk at a constant rate for each trial.

5. Measure the distance the student walked after 2, 4, 6, 8 and 10 seconds.

6. Make sure students are walking at the same pace each time they walk.

7. Help students with all calculations as needed.

8. Answers for the lab will vary based on student times.

Evaluation

Rubric for Graphs

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Neatness and Attractiveness	Exceptionally well designed, neat, and attractive. Colors that go well together are used to make the graph more readable. A ruler and graph paper (or graphing computer program) are used.	Neat and relatively attractive. A ruler and graph paper (or graphing computer program) are used to make the graph more readable.	Lines are neatly drawn but the graph appears quite plain.	Appears messy and "thrown together" in a hurry. Lines are visibly crooked.
Concepts	Student has a clear understanding of plots and has answered the question effectively.	Student has satisfactory understanding of the major concepts, but has small misunderstandings.	Student has major misunderstandings of the concepts and cannot complete work on his own.	Student does not display understanding of the major concepts or did not complete the assignment.

How Do I "Rate?"

Timer's Name ______________________________

Walker's Name ______________________________

Measurer's Name ______________________________

Today we are going to figure out what your rate of walking is.

Procedure:

1. Decide who has each job. (You can change jobs the second time you do this.)
2. Make sure your group has the How Do I Rate worksheet and a stopwatch.
3. The walker should try to walk this course using an even pace each time. He or she will start at the beginning for each trial.
4. The timer will tell the walker to stop after two seconds. The walker will go to the start again and the timer will stop him or her in four seconds. Follow this procedure for six, eight and 10 seconds.
5. The measurer will write the distance the student walked after two, four, six, eight and 10 seconds.
6. Remind the walker to use the same pace each time.
7. Do the experiment a second time.
8. Calculate the rate.

Time Seconds	Distance Trial 1	Distance Trial 2	Rate x Time = Distance	Rate
2				
4				
6				
8				
10				

What is the independent variable in this experiment (x-axis)?

What is the dependent variable (y-axis)?

Make a graph using data from Trial 1 *or* Trial 2.

NOTE: Did you use a title? Did you label the axes? Do you have consistent intervals?

If your measurements were accurate, you should be able to connect the dots in a straight line. Study the points and make a straight line that looks like it might be an accurate representation of the data.

Write two sentences that explain your graph.

1.

2.

Name______________________________

Summative Evaluation

1. What was the rate and speed of your group's car?

2. What caused the differences of rate and speed between groups?

    a. What if they used the same ramp?

3. How did your graph increase as the time changed?

4. When you created your graph, describe the line that your data created.

    a. What made the line stay constant?

    b. Why did the graph start at the origin?

5. Whose car went the furthest?

    a. What about the experiment made their car go the furthest?

6. Compared to the Indy car, how many times faster/slower did your car travel?

7. How much further behind would your car be if it and the Indy car had a race that lasted for 12 seconds?

8. How much time would it take for your car to travel one mile (5,280 ft.)?

9. How much time would it take the Indy car to travel one mile (5,280 ft.) at 100 mph?

Graphing Linear Equations Vocabulary

Binomial: A polynomial of only two terms.

Coefficient: The numeric factor in a term; e.g., the number 3 in the term 3*x*2*y* is the coefficient. In the term *a*3*b*, 1 is the coefficient.

Coordinate plane: A plane in which an ordered pair can be located by reference to two intersecting number lines.

Coordinate of a point: The number paired with that point on a number line.

Coordinates of a point: See ordered pairs.

Constant: A monomial consisting of a numeral only; a term with no variable factor.

Function: A mathematical relationship between two variables, an independent variable and a dependent variable, where every value of the independent variable corresponds to exactly one value of the dependent value.

Inequality: A mathematical sentence that includes one of the inequality symbols <, >, ≤, ≥, or ≠ to compare unequal expressions.

Equation: A statement that shows two mathematical expressions that are equal to each other.

Horizontal axis: The horizontal number line in a coordinate plane, also called the x-axis.

Linear equation: An equation whose graph on a coordinate grid is a straight line.

Monomials: An algebraic expression which is a product of constants and *variables*.

Ordered pairs: A pair of numbers for which the order of the numbers is important. (2, -3) is an ordered pair.

Polynomials: The sum of monomials; e.g., 2*a*2 + 4*a* – 5.

Quadrants: The two axes of a coordinate system divide the plane into four separate sections known as quadrants. These are identified as the first, second, third, and fourth quadrants.

Slope of a line: If (x_1, y_1) and (x_2, y_2) are two different points on a line, the slope of the line is given by

$$\frac{y_2-y_1.}{X_2-y_2}$$

A horizontal line has a slope of 0 and a vertical line has no slope.

Slope-intercept form of an equation: The equation of a line in the form $y = mx + b$, where m is the slope and b is the y-intercept.

Solution of a sentence: Any value of a variable that turns an open sentence into a true statement.

Solve an equation: To find the set of all solutions of the equation.

Standard form of a linear equation: $ax + by = c$, where a, b and c are integers and a and b are not both zero.

Univariate data: Having one variable.

Variable: A changing quantity, usually a letter in an algebraic equation or expression, that might have one of a range of possible values.

Vertical axis: The vertical number line in a coordinate plane.

X-axis: The horizontal axis in the coordinate plane.

X-intercept: The x-coordinate of a point where a graph intersects the x-axis.

Y-axis: The vertical axis or a coordinate plane

Y-intercept: The y-coordinate of a point where a graph crosses the *y*-axis.

math in Motion

Inequalities

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Chapter Synopsis

The coordinate plane is a two-dimensional surface on which we can plot points, lines and curves. It has two scales, called the x-axis and y-axis, at right angles to each other that meet at the point of origin (0, 0). In this module, students use the concept of Formula M racing to understand the concept of inequalities. What does it mean if a solid line is found on the plane and all areas below the line are shaded? This could mean that anything on the line is a tie in the race and anything in the shaded area would be a loss. In other words, we're looking at real-life activities to teach this abstract concept.

Lesson one has students using rates to calculate the lowest rate needed to beat a world record. This lesson focuses on using the slope-intercept form of an equation to help students solve an inequality by creating a table and coordinate graph to represent the situation.

In the second lesson, students first work with different inequality symbols. They then use rates to write linear equations to calculate all possible solutions for different situations. This lesson focuses on using the slope-intercept form of an equation to help students solve an inequality with inverse operations and represent all solutions on a coordinate graph.

Formative and summative assessments are available, plus a resource sheet on inequalities and a vocabulary list.

Standards Addressed

Grade 8, Mathematics — Patterns, Functions and Algebra

08-10 Benchmark F. Solve and graph linear equations and inequalities.

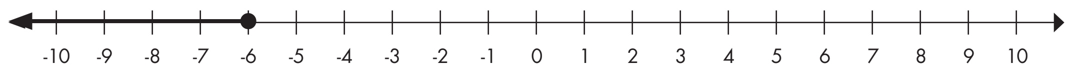
Y2003.CMA.S04.G08-10.BF.L08.107 / Use Algebraic Representations

07. Use symbolic algebra (equations and inequalities), graphs and tables to represent situations and solve problems.

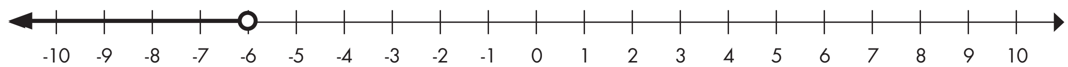
Inequalities

A number line shows the order relationships among real numbers. If you have a number line and the number 2, you can show that by putting a point at 2. You can also show inequalities on a number line. Math problems containing <, >, $\leq$=, and $\geq$= are called inequalities.

If you want to show x $\leq$ = -6 the number line would look like this:



If you want to show x < (but not equal to) -6, you make the arrow go in the same direction, but the circle at -6 would be an empty circle.



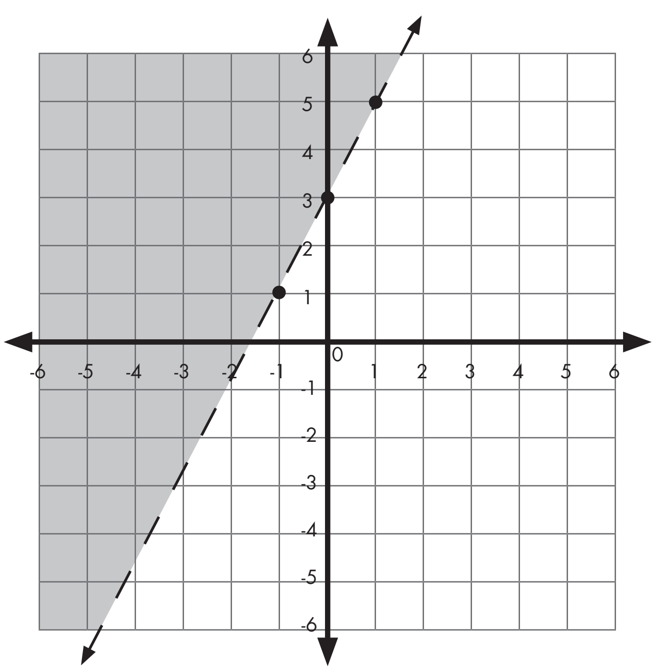
Solving an inequality is very similar to solving an equation. You follow the same steps, except for one very important difference. When you multiply or divide each side of the inequality by a negative number, you have to reverse the inequality symbol!

$$-2x > 10 \quad = \quad \frac{\cancel{-2}x}{\cancel{2}} > \frac{10}{-2} \quad = \quad x < -5$$

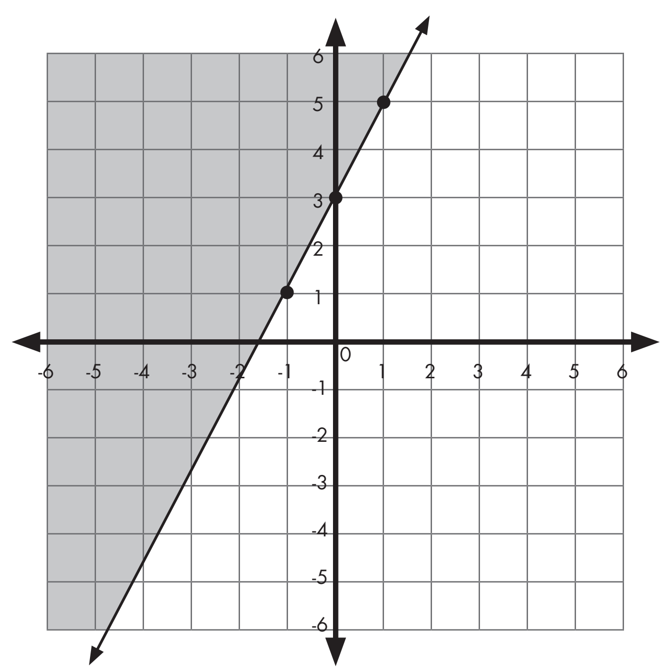
Why do we reverse the symbol? Let's see what happens if we don't. Think about the simple inequality –2 < 6. This is obviously a true statement. If we divided both sides by -2, we would get 1 < -3, which is not true! The inequality must be 1 > -3.

Linear relationships can also be inequalities. The equation of this line is **y = 2x + 3.** If this were the inequality y > 2x + 3, the area above the line would be shaded because all coordinates above the line will make the inequality true. The line itself would be broken because those coordinate would not make the statement true. If the sentence said y >= 2x +3, all points above the line would be shaded and the line would be solid. This is because the points on the line also make the equation true.

y > 2x + 3



y $\geq$ = 2x +3



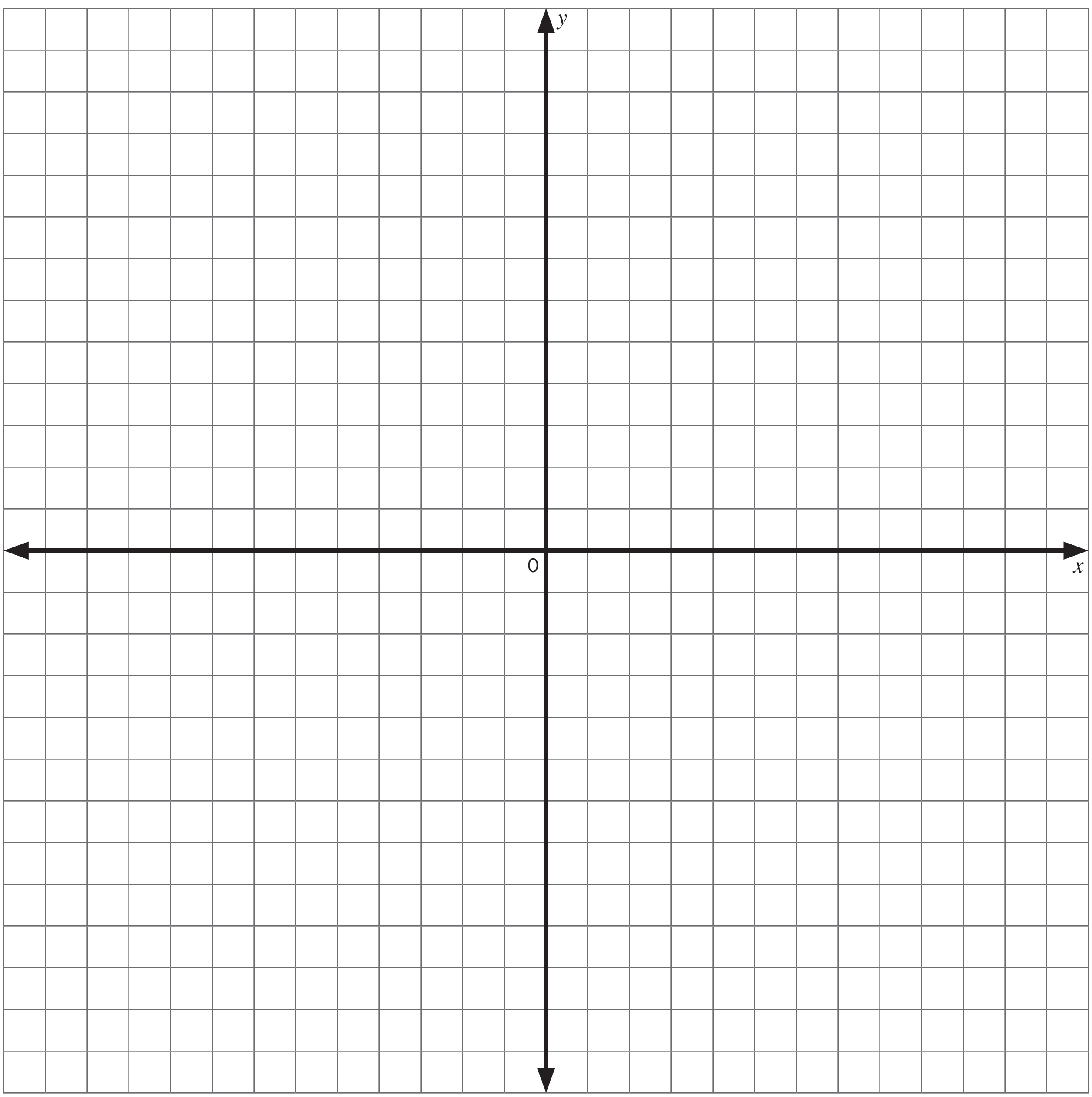
Graphing Inequalities
Formative Evaluation

1. The inequality y ≥ x + 7 would have a ____ line.
(Choose solid or broken)

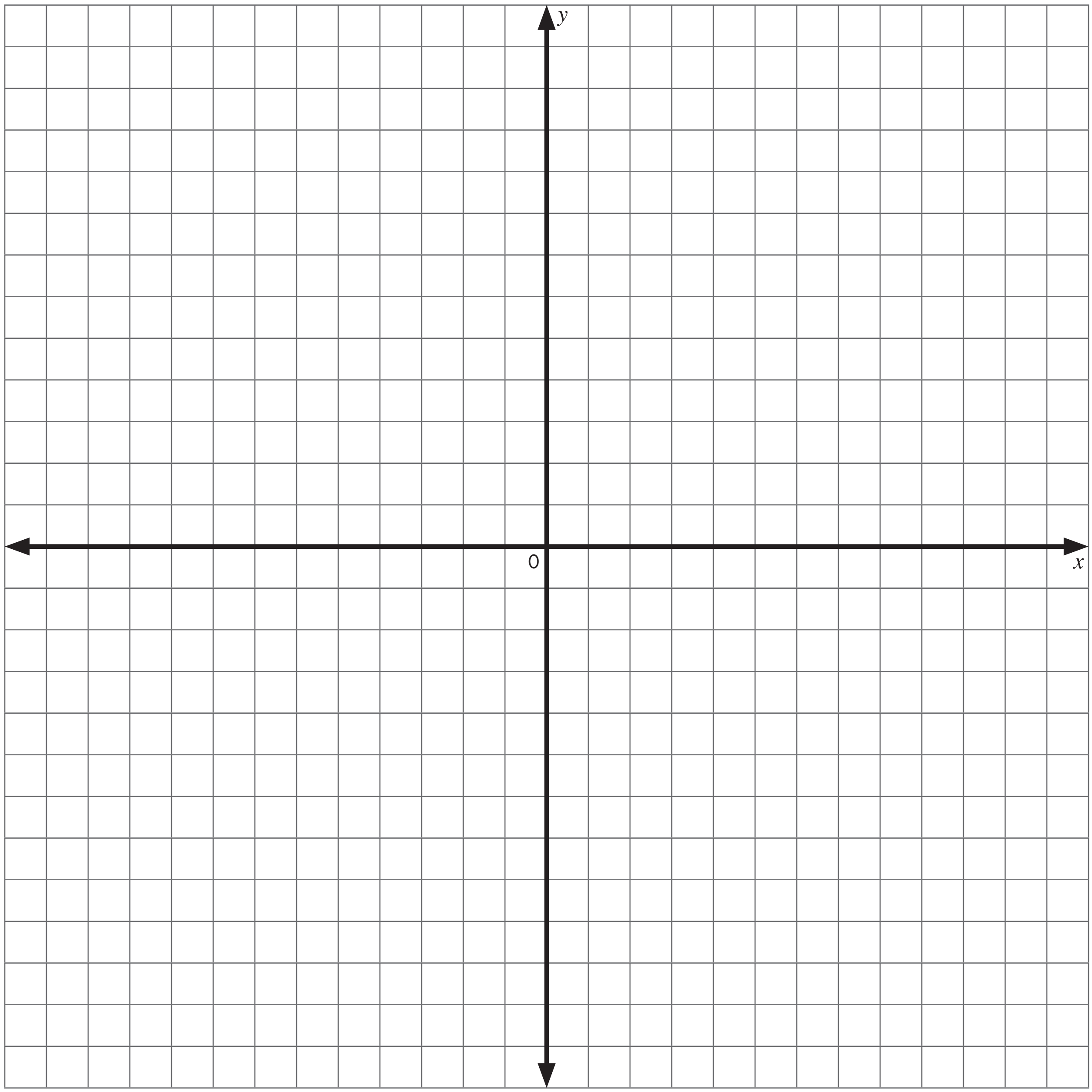
2. The inequality 2x – 1 < y would have a ____ line.
(Choose solid or broken)

3. Graph the following inequalities.

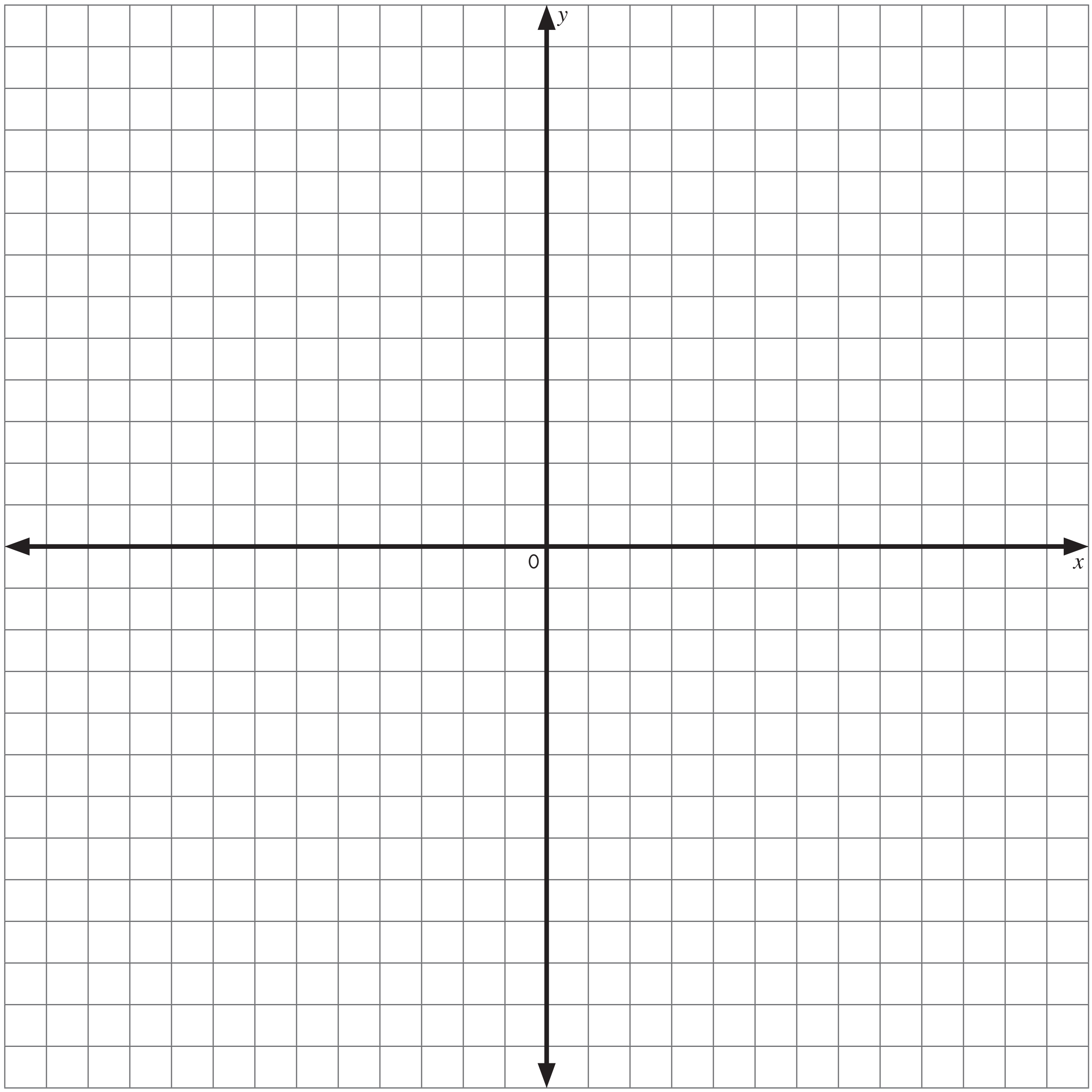
   a. $x + 2 \geq y$



b. $y < -2x + 1$



c. $y \geq 4x + 3$



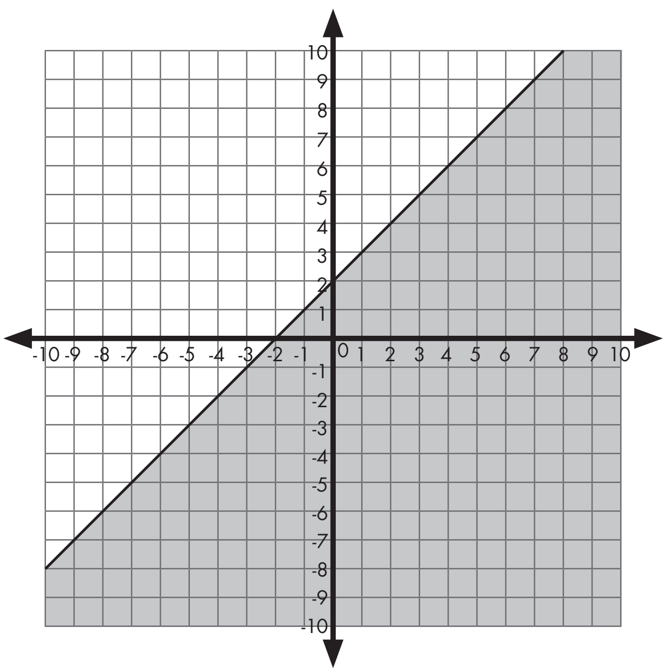
Answer Key: Graphing Inequalities
Formative Evaluation

1. The inequality $y \geq x + 7$ would have a ____solid____ line.

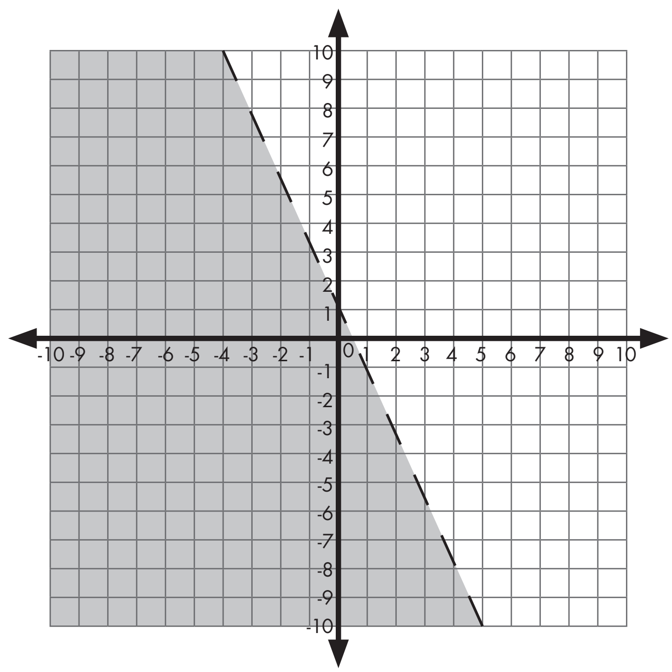
2. The inequality $2x - 1 < y$ would have a ____broken____ line.

3. Graph the following inequalities.

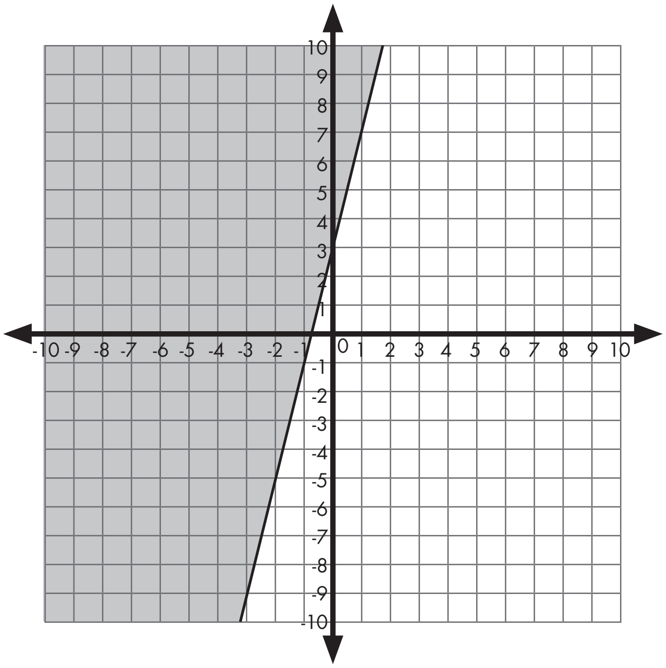
a. $x + 2 \geq y$



b. $y < -2x + 1$



c. $y \geq 4x + 3$



Solving Inequalities

Overview

Students use rates to calculate the lowest needed rate to beat a world record. This lesson focuses on using the slope-intercept form of an equation to help students solve an inequality by creating a table and coordinate graph to represent the situation.

Standards Addressed

Grade 8, Mathematics — Patterns, Functions and Algebra

08-10 Benchmark F. Solve and graph linear equations and inequalities.

Y2003.CMA.S04.G08-10.BF.L08.I07 / Use Algebraic Representations

07. Use symbolic algebra (equations and inequalities), graphs and tables to represent situations and solve problems.

08-10 Benchmark D. Use algebraic representations, such as tables, graphs, expressions, functions and inequalities, to model and solve problem situations.

Y2003.CMA.S04.G08-10.BD.L08.I07 / Use Algebraic Representations

07. Use symbolic algebra (equations and inequalities), graphs and tables to represent situations and solve problems.

Y2003.CMA.S04.G08-10.BD.L08.I08 / Use Algebraic Representations

08. Write, simplify and evaluate algebraic expressions (including formulas) to generalize situations and solve problems.

Materials

- Graphing calculator or other random number generator

Procedure

1. Discuss with the class what it means to do something at a steady pace or constant rate. What happens if you increase the pace? How would the solutions compare to the original pace? Introduce the inequality symbols and have a class discussion about how each of the symbols may be read: more than (>), and less than (<). The solid line below the symbol signifies greater than or equal to (≥) or less than or equal to (≤).

2. Pass out the handout with stars on it and blank paper. Explain to the students that they are going to find the rate that they can trace the stars and compare their rate to the fastest student from previous years. They must produce as many neat, complete tracings of the stars as possible in one minute.

3. Once the time is up, have the students count up their total stars completed. Since there are 10 line segments per star, the partial portion converts easily into a decimal.

4. Distribute the Star Tracing Exercise handout. Have each student write a linear equation to find the total stars they traced for any number of seconds. Tell students that they are going to try to find out what pace they need to keep in order to match or beat your fastest student. Have students complete the table for the record holder and write a linear equation to find the total number of stars for any time in seconds.

5. Use a graphing calculator to have students generate 10 pairs of random numbers. If a calculator is not easily accessible, writing possible numbers on index cards and pulling one for x and one for y would also be sufficient. This helps the students look at the rates for specific points to determine if they are a solution to the inequality.

6. Point out to the class that since you want to beat the record and not tie the record, the line should be a dotted line to show that it must be greater than and cannot be equal to the record holder's pace.

Evaluation

After completing the stars activity, have students select one of the Guinness Book of World Records scenarios to graph all possible solutions.

Category	Greater Than	Equal to	Less Than
Identifying Correct Solution Set on Coordinate Grid	The line is correctly marked as dashed or solid. Appropriate region is shaded.	The line is incorrectly marked or an area is incorrectly shaded.	Neither the line nor the solution set is shaded appropriately.
Writing Algebraic Inequality	Algebraic inequality is written correctly, with the correct rate and inequality symbol.	Either the algebraic inequality is written incorrectly or the correct rate and inequality symbol are not used.	Neither the inequality nor the symbols are correct.
Correctly Identifying Five Possible Solutions	Both the x and y values are correctly written in the table, and all five values are solutions.	Both the x and y values are correctly written in the table, and three or four values are solutions.	Both the x and y values are correctly written in the table, and one or two values are solutions.

Can You Beat It?

1. My rate: ________________ stars per minute OR ________________ stars per second

   My equation for total stars, *s*, in *t* seconds is: ________________

2. The record holder's is 750 stars in five minutes.

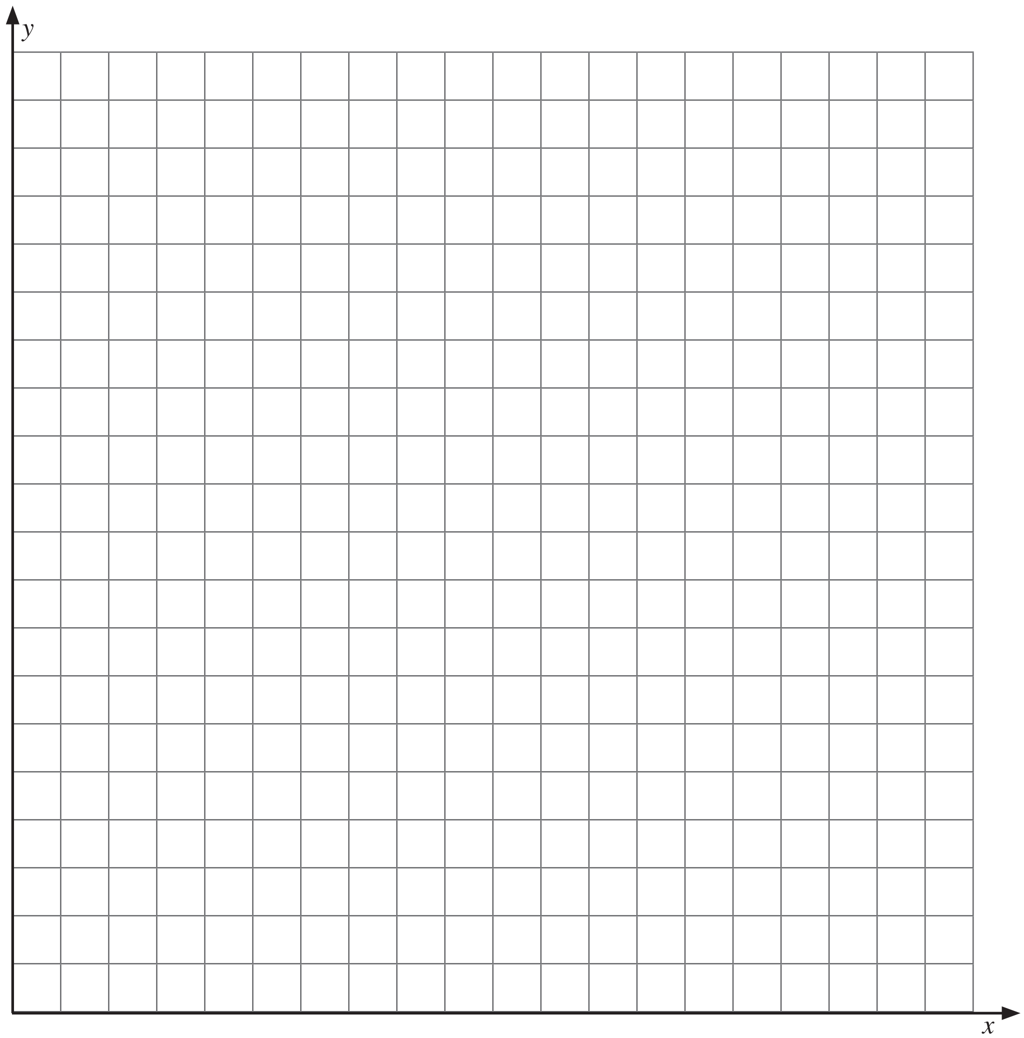
   ________________ stars per minute OR ________________ stars per second

   The equation for total stars, *s*, in *t* seconds is: ________________

3. Compare your rate to the record holder:

Time in minutes	***Time in seconds***	***Total Stars:*** ***Me***	***Record Holder***
0			
1			
2			
5			
10			
15			
60			

4. In order to beat the record holder, I must have a rate of more than ________________ stars per second, so the inequality is written as ________________________.

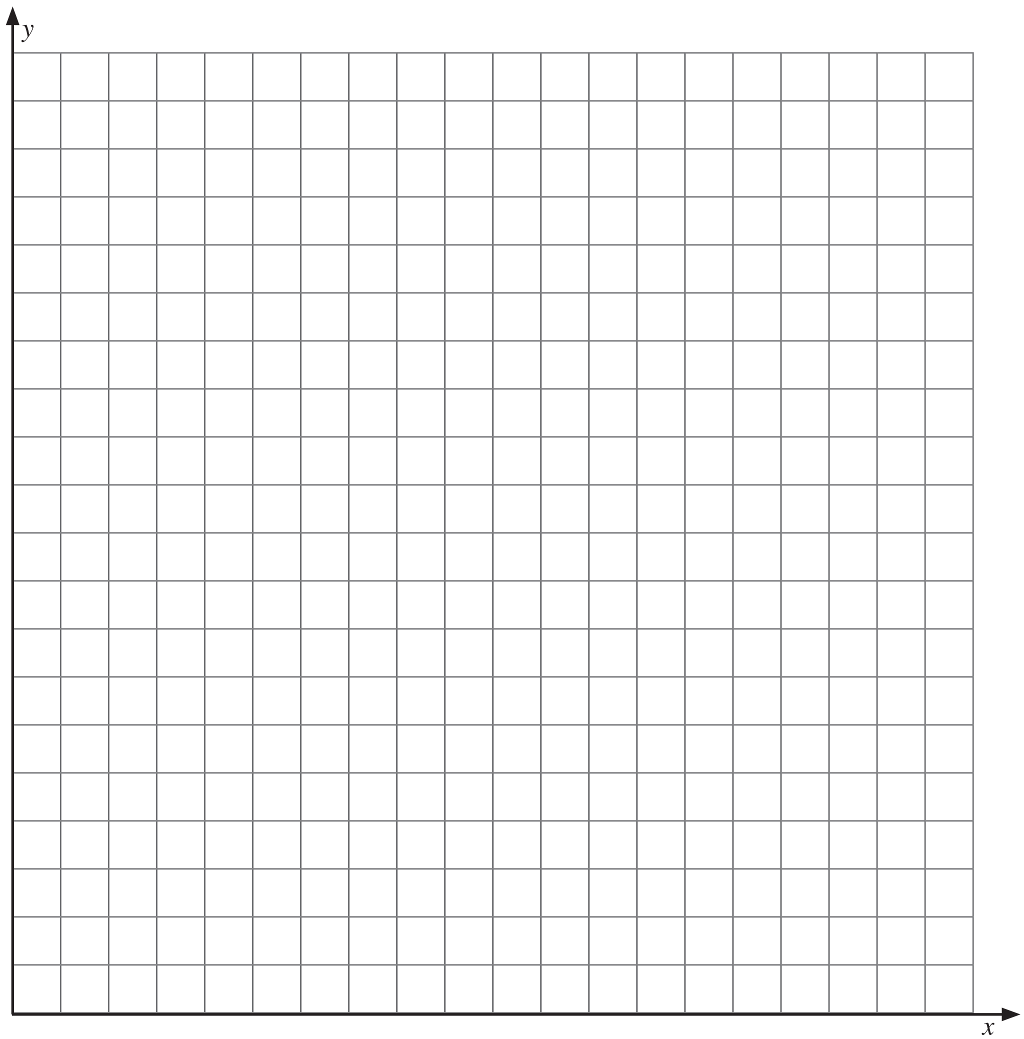


5. Use a graphing calculator to produce 10 random coordinate pairs. Record the pairs, calculate the pace or the rate and determine if it is a record-setting pace!

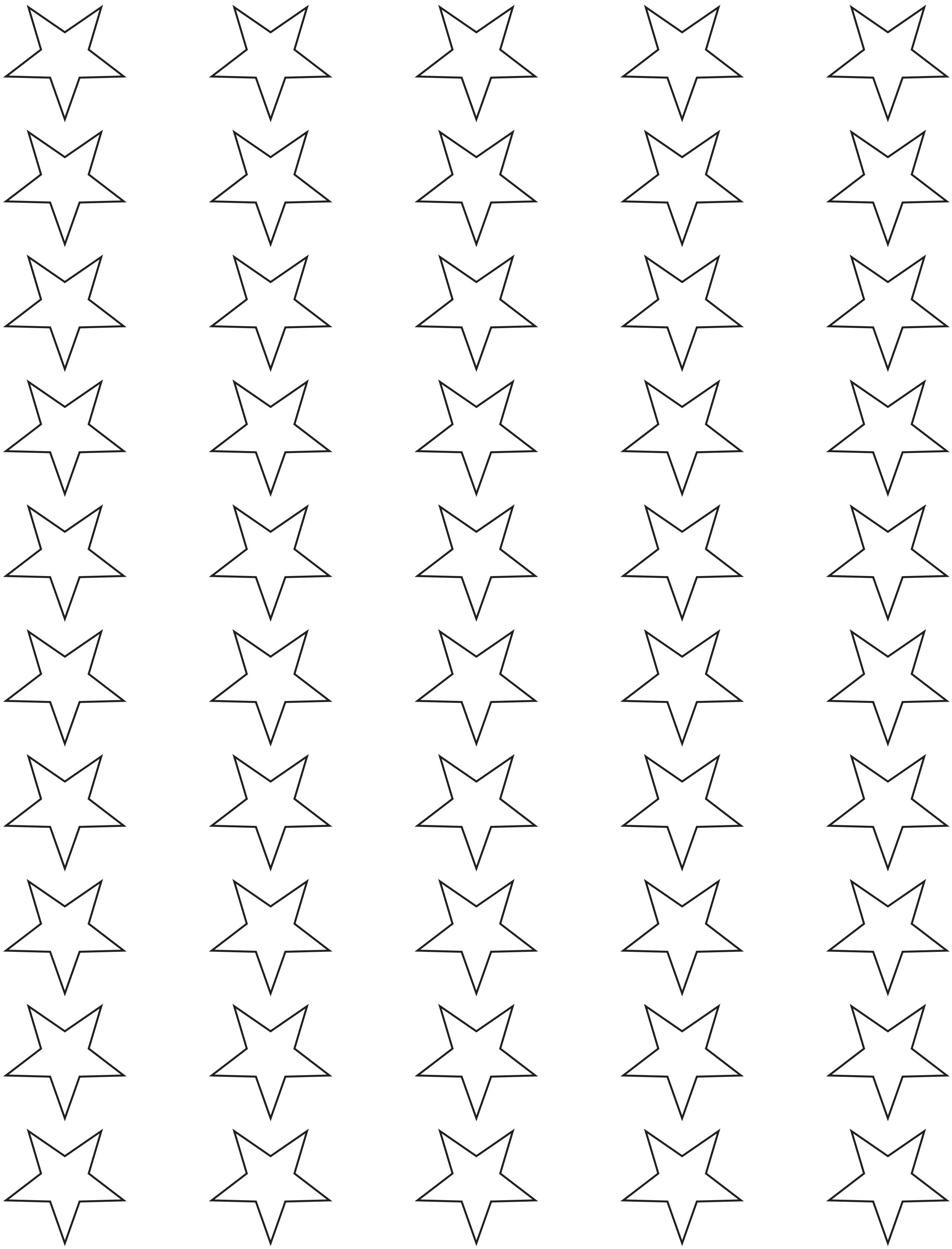
Time in Seconds	***Total Stars***	***PACE (stars per second)***	***Faster, Slower or Equal to?***

6. On the coordinate grid, graph the line that represents the inequality to beat the record holder. Should it be dashed or solid?

7. Shade in the portion of the graph that shows all possible rates that would allow you to break the record. Be sure to title your graph and label the axes.



Stars Handout



World Records

Fastest time to pop 1,000 balloons: 37 participants popped 1,000 balloons in one minute, 44 seconds.

Most heads shaved in one hour by a team: 10 hairdressers shaved 56 head in one hour.

Longest supermarket cart push: a team of 801 people pushed a cart 1,098 miles in 36 days.

Most trees planted in an hour: 4,100 trees in one hour.

Most tornadoes in 24 hours: 148 tornadoes.

Longest lawnmower ride: 14,594 miles in 260 days.

Longest roller coaster marathon: 192 hours.

Most roller coasters ridden in 24 hours: 74.

Fastest wooden roller coaster: 78.3 mph.

Most Smarties eaten in three minutes: 112.

Most cherry stems knotted in three minutes: 3.9.

Fastest time to eat a raw onion: two minutes, 40 seconds.

Most cooked brussels sprouts eaten in one minute: 43 sprouts.

Most baked beans eaten in five minutes: 226.

Most darts bull's-eyes in 10 hours: 1,432.

Most pumpkins carved in one hour: 42.

Fastest growing tree: Albizzia falacata, 35 ft 2.8 in in 13 months or 1.1 in per day.

Fastest land speed: 763.03 mph.

Source: Guinness World Records 2005, HIT Entertainment 2004

Graphing Inequalities

Overview

Students begin by doing an activity that uses inequality symbols. They then will use rates to write linear equations to calculate all of the possible solutions for different situations. This lesson focuses on using the slope intercept form of an equation to help students solve an inequality with inverse operations and representing all solutions on a coordinate graph.

Standards Addressed

Grade 8, Mathematics — Patterns, Functions and Algebra

08-10 Benchmark F. Solve and graph linear equations and inequalities.

Y2003.CMA.S04.G08-10.BF.L08.I07 / Use Algebraic Representations

07. Use symbolic algebra (equations and inequalities), graphs and tables to represent situations and solve problems.

08-10 Benchmark D. Use algebraic representations, such as tables, graphs, expressions, functions and inequalities, to model and solve problem situations.

Y2003.CMA.S04.G08-10.BD.L08.I07 / Use Algebraic Representations

07. Use symbolic algebra (equations and inequalities), graphs and tables to represent situations and solve problems.

Y2003.CMA.S04.G08-10.BD.L08.I08 / Use Algebraic Representations

08. Write, simplify and evaluate algebraic expressions (including formulas) to generalize situations and solve problems.

Materials

- Blank coordinate grids
- Graphing calculators (TI-73 or TI-83)

Procedure

1. Discuss as a class the different inequality symbols and how they sound in questions (i.e. greater than, at least, less than and equal to).

2. Divide students into pairs and give each pair one set of inequalities graphs plus one copy of the Introduction to Inequalities handout. Have the pairs separate the graphs into five categories of graphs that show equation values as follows:

    a. greater than

    b. greater than or equal to

    c. equal to

    d. less than

    e. less than or equal to

6. Discuss the categories and where each graph best fits. Instruct the students to pay close attention to the type of line (dashed or solid).

7. Review how to write linear equations in the slope-intercept form, and how to solve for the independent variable using inverse operations.

8. Give each pair of students the Speeds handout and instruct them to write an inequality for each situation. Then have the students graph the inequalities on a coordinate grid or graphing calculator.

    Note that the graphing calculator always uses a solid line. Students will have to know if the line should be solid or dashed.

9. Discuss the inequalities, solutions and graphs as a class. Explain that it is important to know if the line is solid or dashed. One way to check if the correct section on the graph is shaded is to select a coordinate point from the section and substitute it into the inequality. If the point makes the inequality true, then the point should fall in the shaded section. If it makes the inequality false, then it should be unshaded.

Evaluation

Students need to have the correct type of line and inequality symbol, plus they should shade in the correct portion of the graph on the student sheet Graphing Inequalities. Use the following chart for evaluation:

Question	**Possible**	**Points Given**
1	2	
2	2	
3 Type of line	2	
3 Correct symbol	2	
3 Correct area shaded	2	
4 Explanation	4	

Enrichment

For additional practice, give students blank coordinate grids and 10 linear equations (e.g. y ___ 2x + 3). Have the students roll an eight-sided die with two sides of each of the following: "less than," "greater than," "greater than or equal to" and "less than or equal to." Students must then use the inequality to complete the equation, graph it and shade in all the possible solutions on a grid.

Name(s) ______________________________

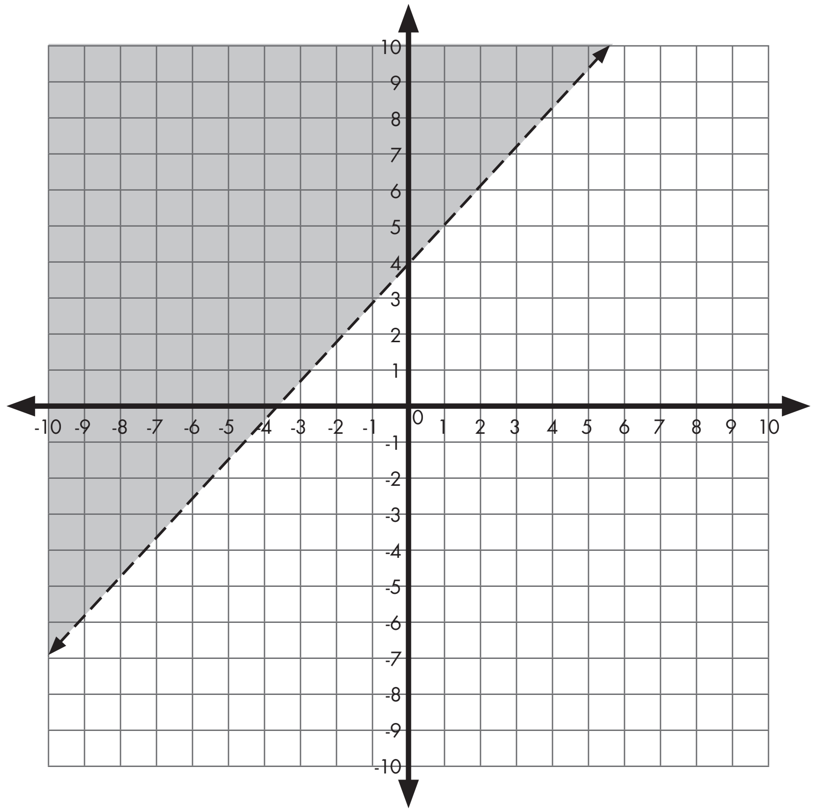
Introduction to Inequalities

Types of Inequalities

	>	≥	<	≤
Words How does it sound?				
Coordinate Graph What does it look like?				

Inequalities Flash Cards

#1



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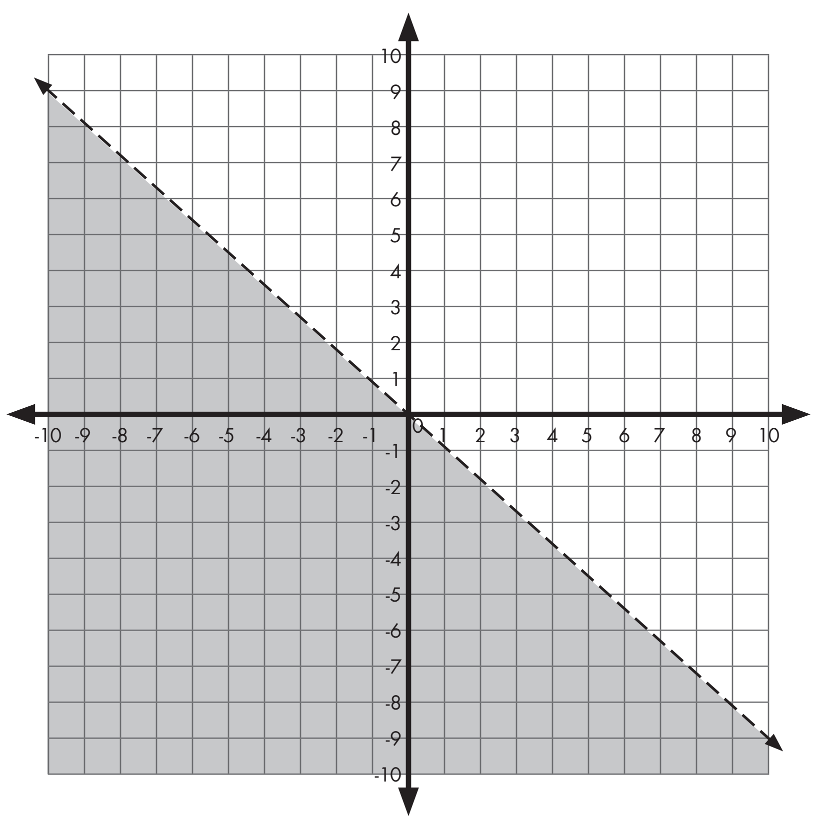

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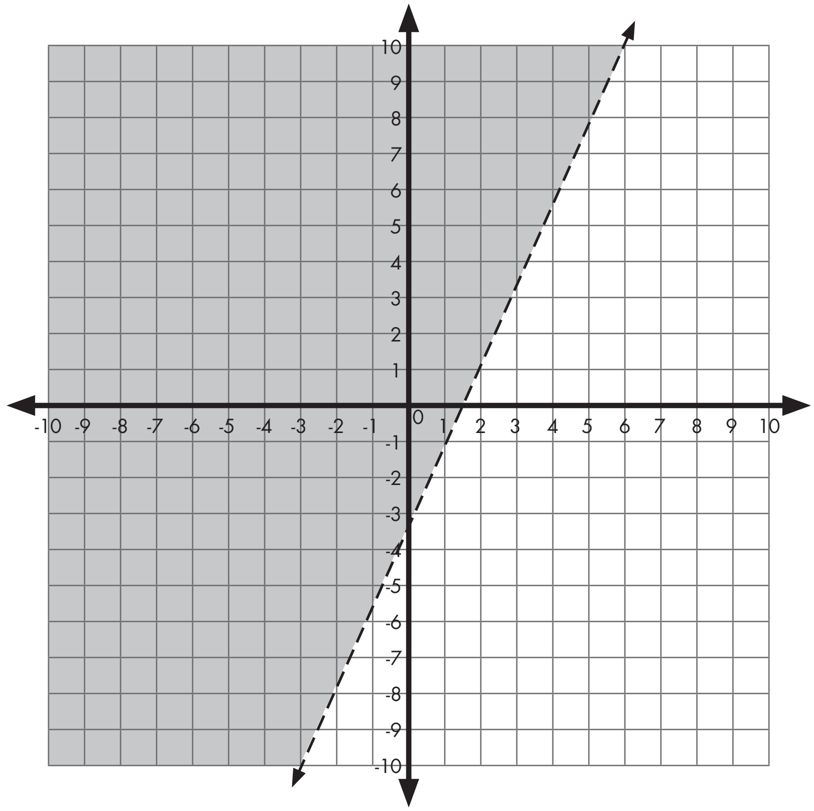
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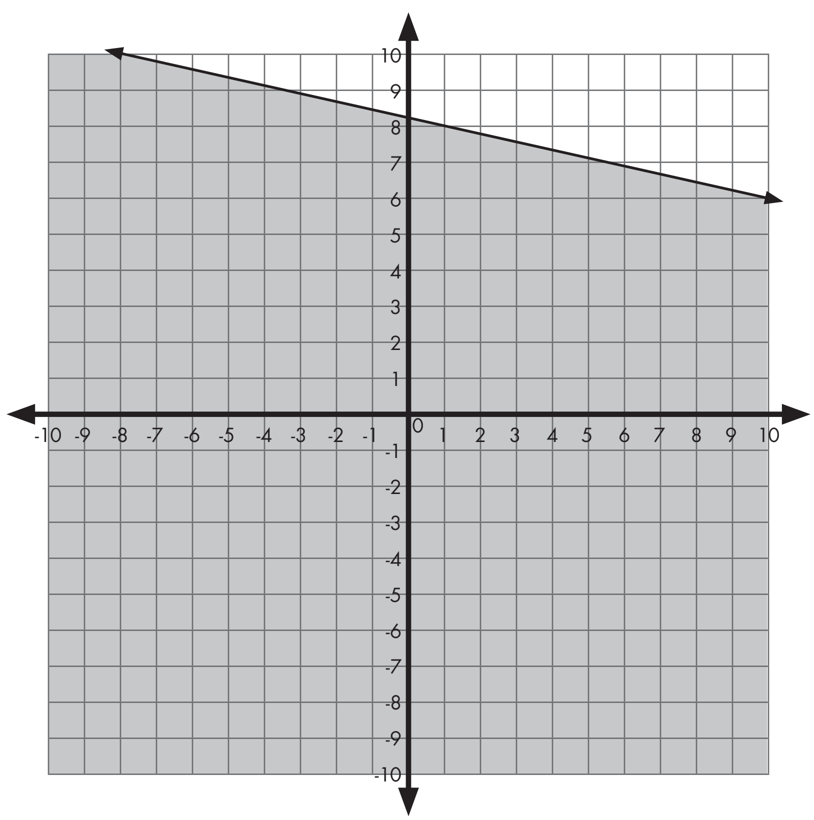
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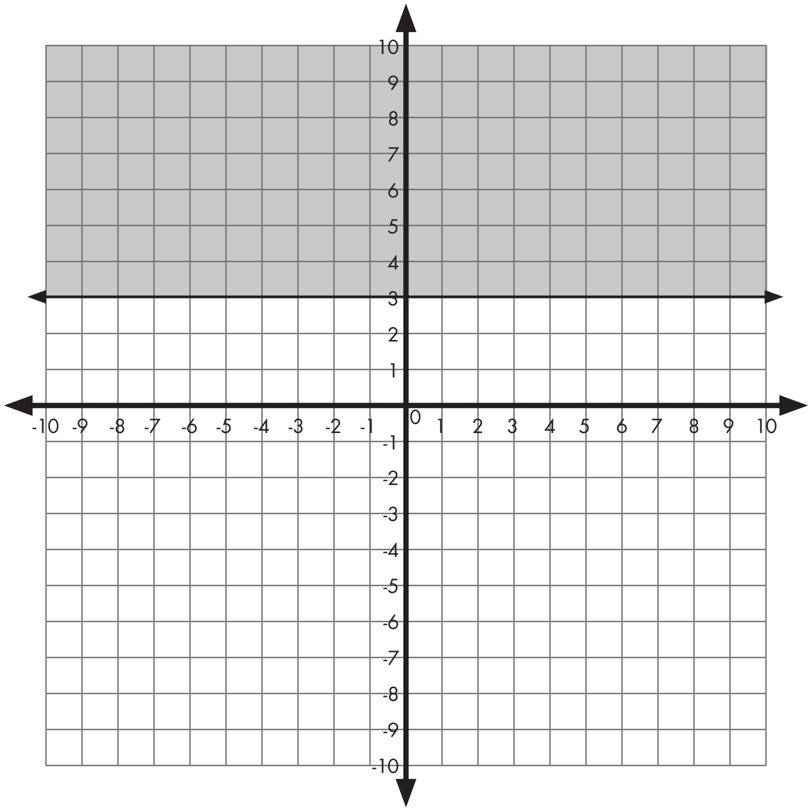

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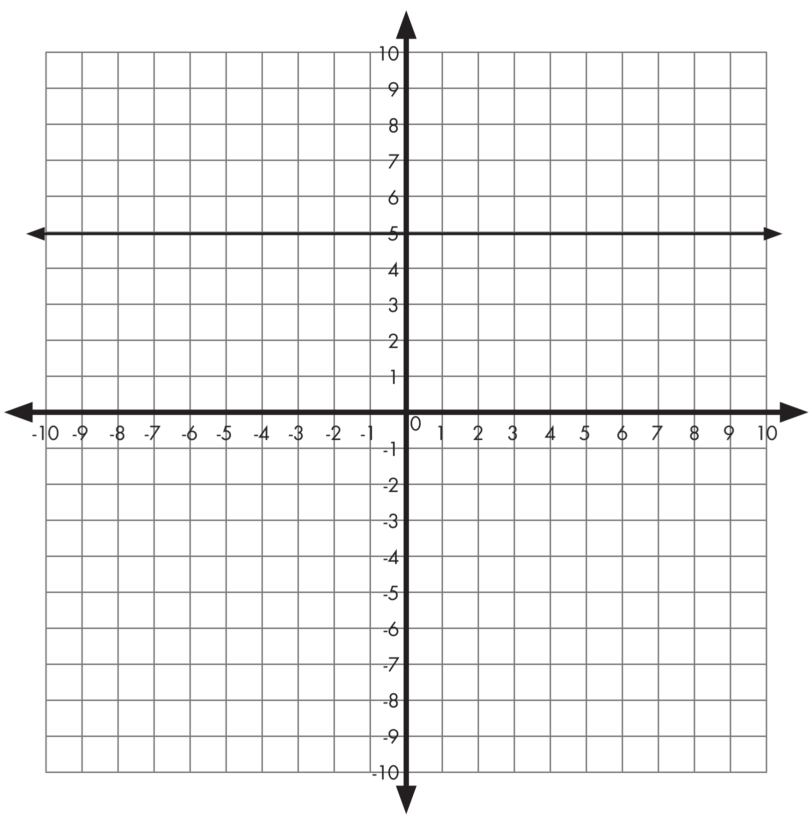

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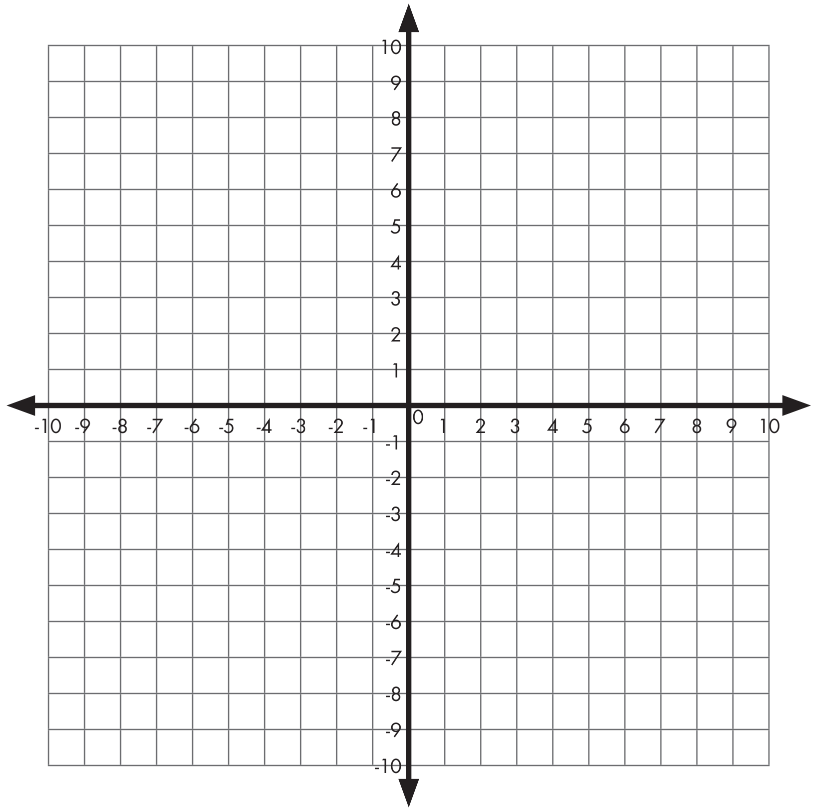
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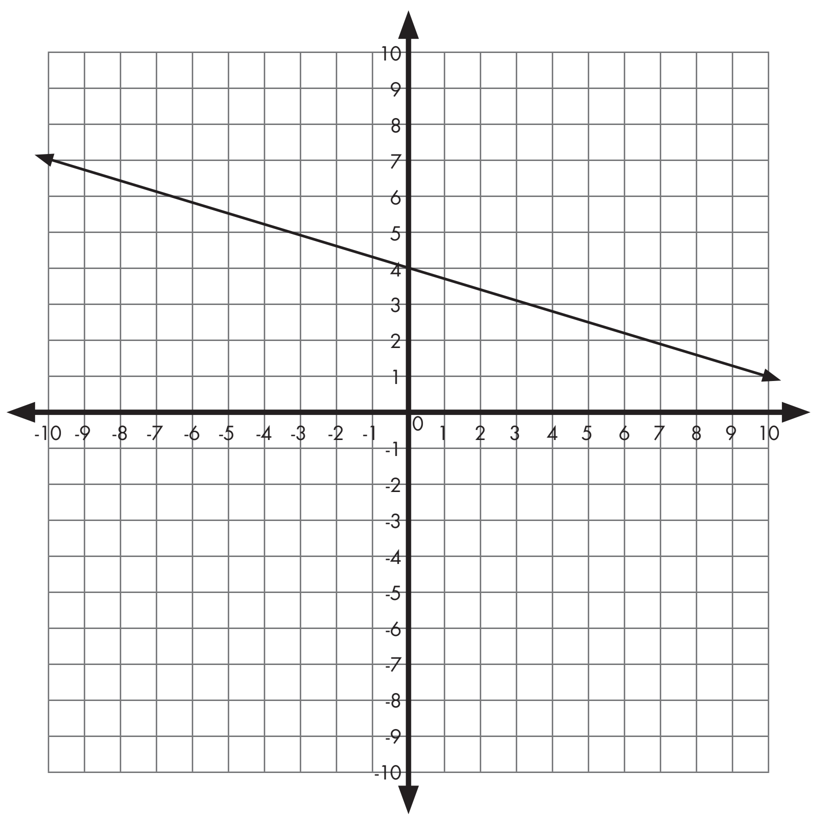
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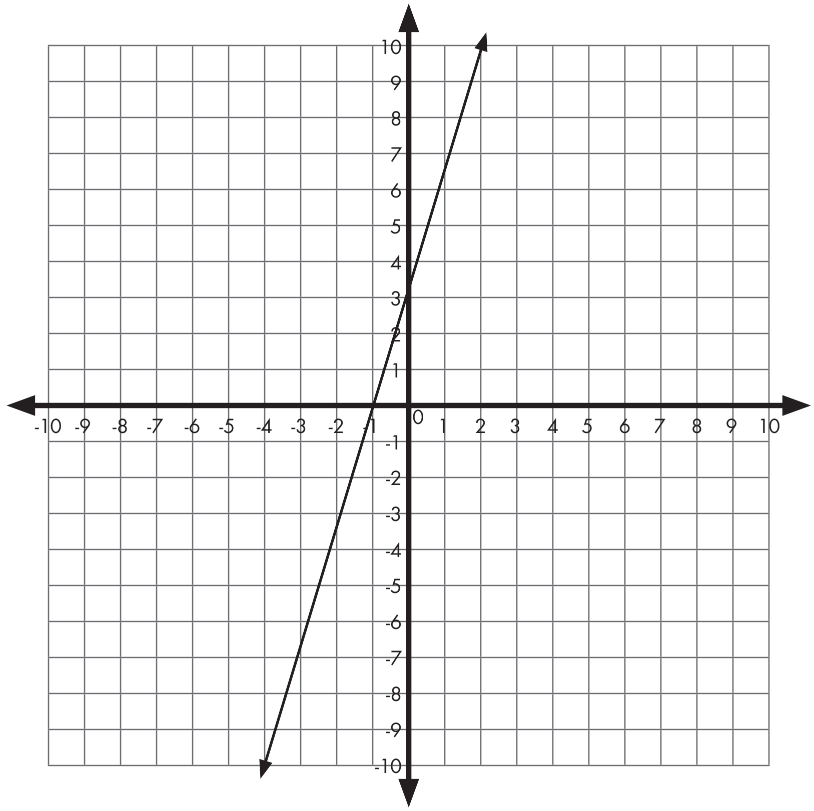

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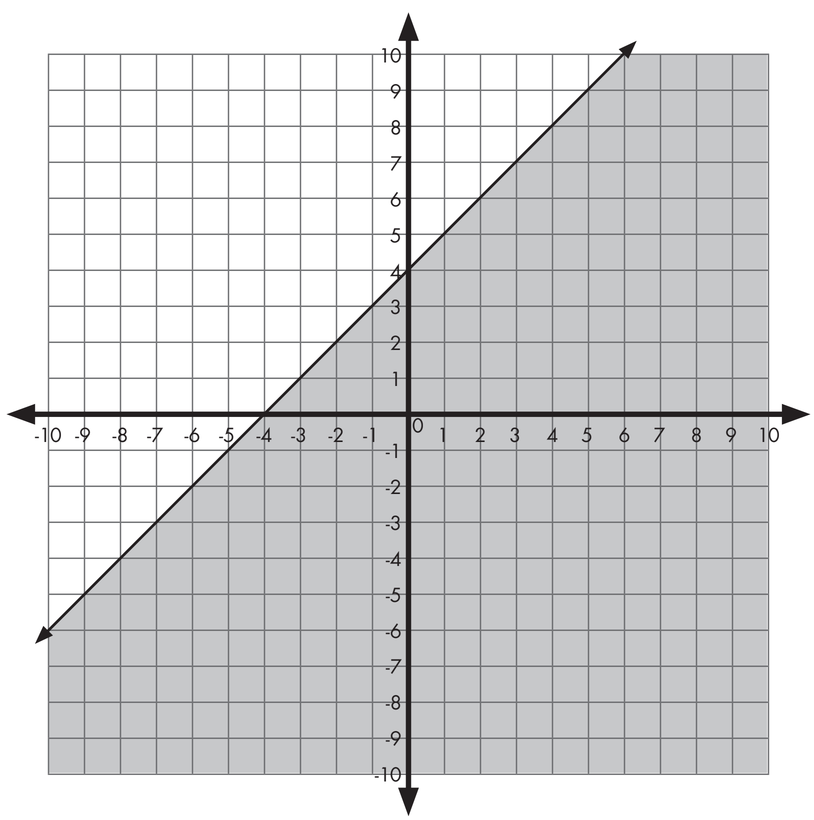

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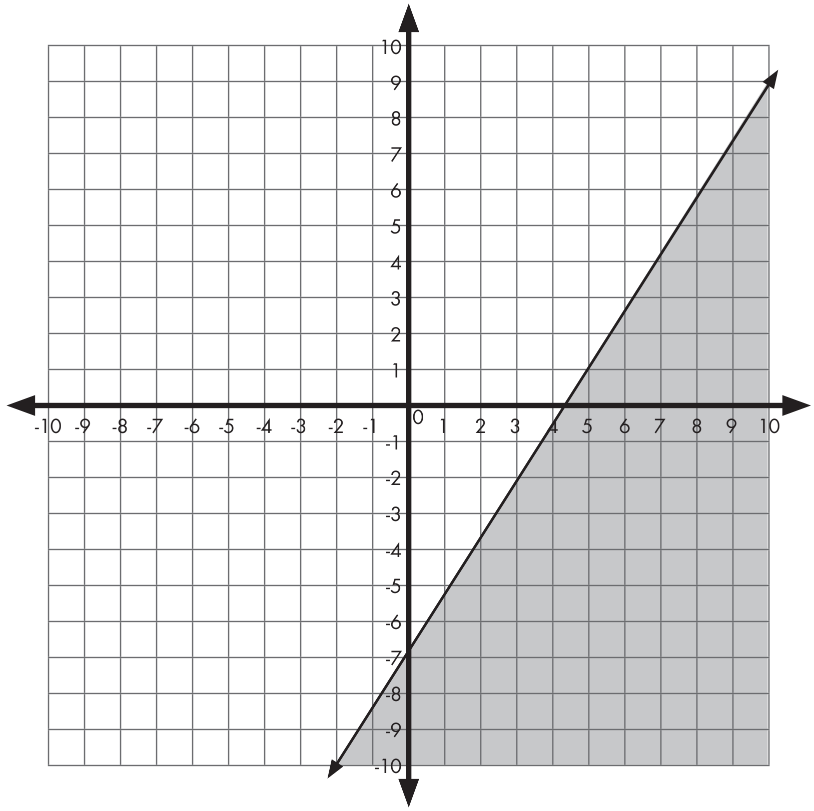

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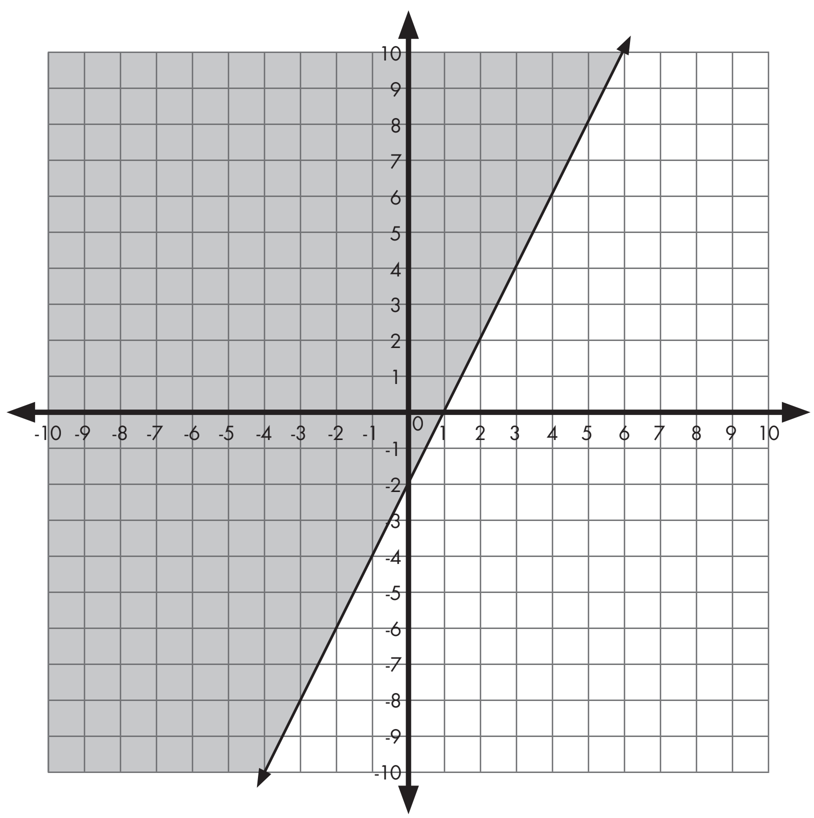

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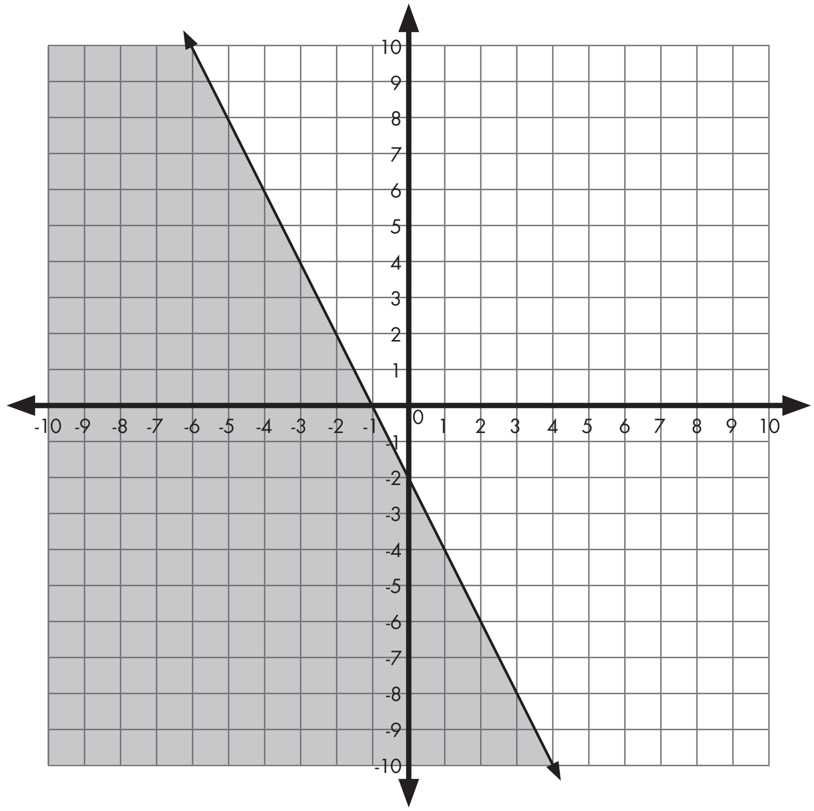


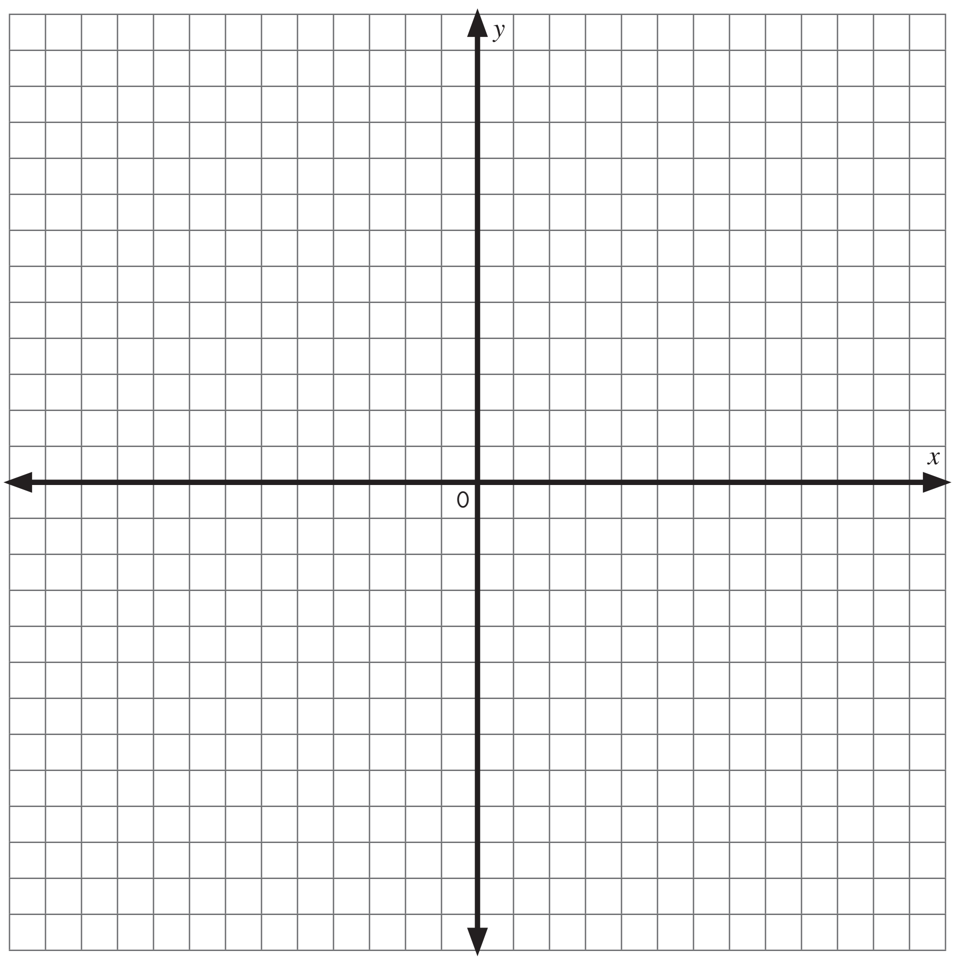
Name(s) ____________________

Speeds

What are all the possible speeds for the losers?

If the winning car had an average speed of 105 mph, how many feet per second did he travel?

1. The equation to find the distance (in feet) traveled in any number of seconds (s) is ____________________
2. The inequality to find all the possible speeds for the losers is ____________________
3. Graph all possible speeds on the coordinate grid below.



4. Does it make sense to graph in Quadrants 2, 3, and 4? Explain.

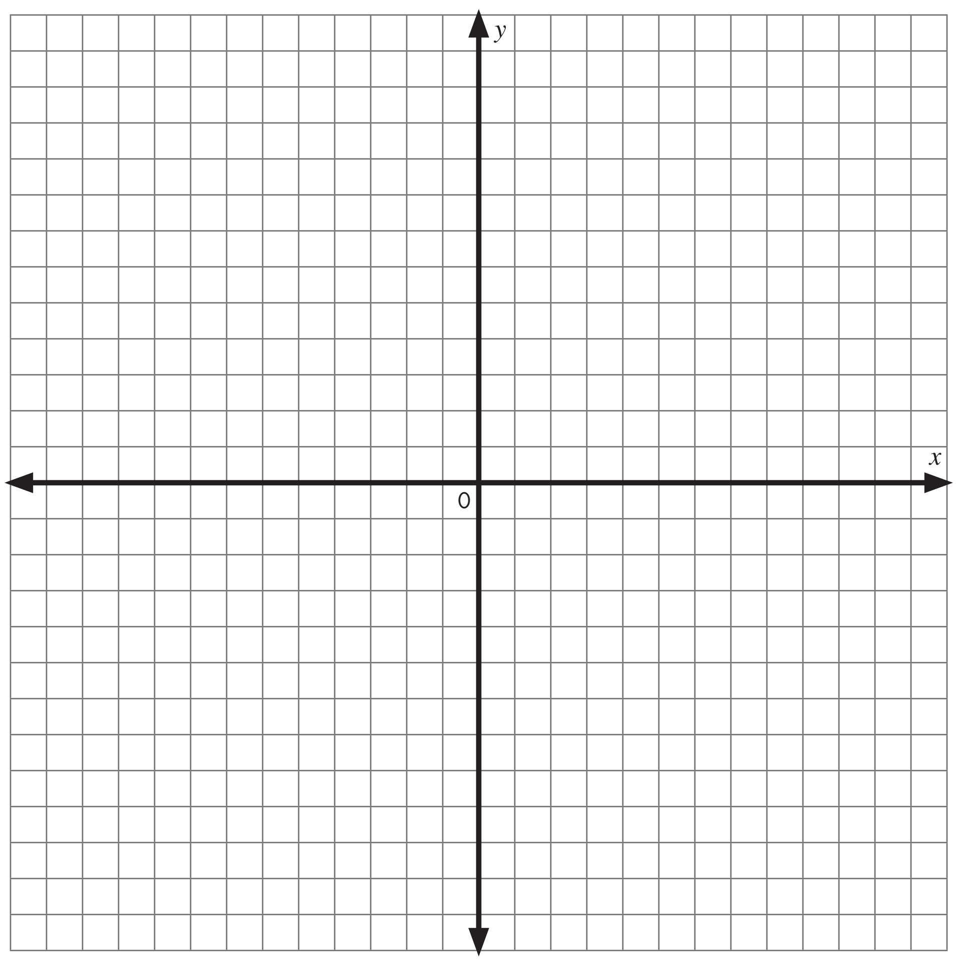
Graphing Inequalities
Summative Evaluation

1. The inequality $y < 2x - 2$ would have a ____________________ line.
(choose broken or solid)

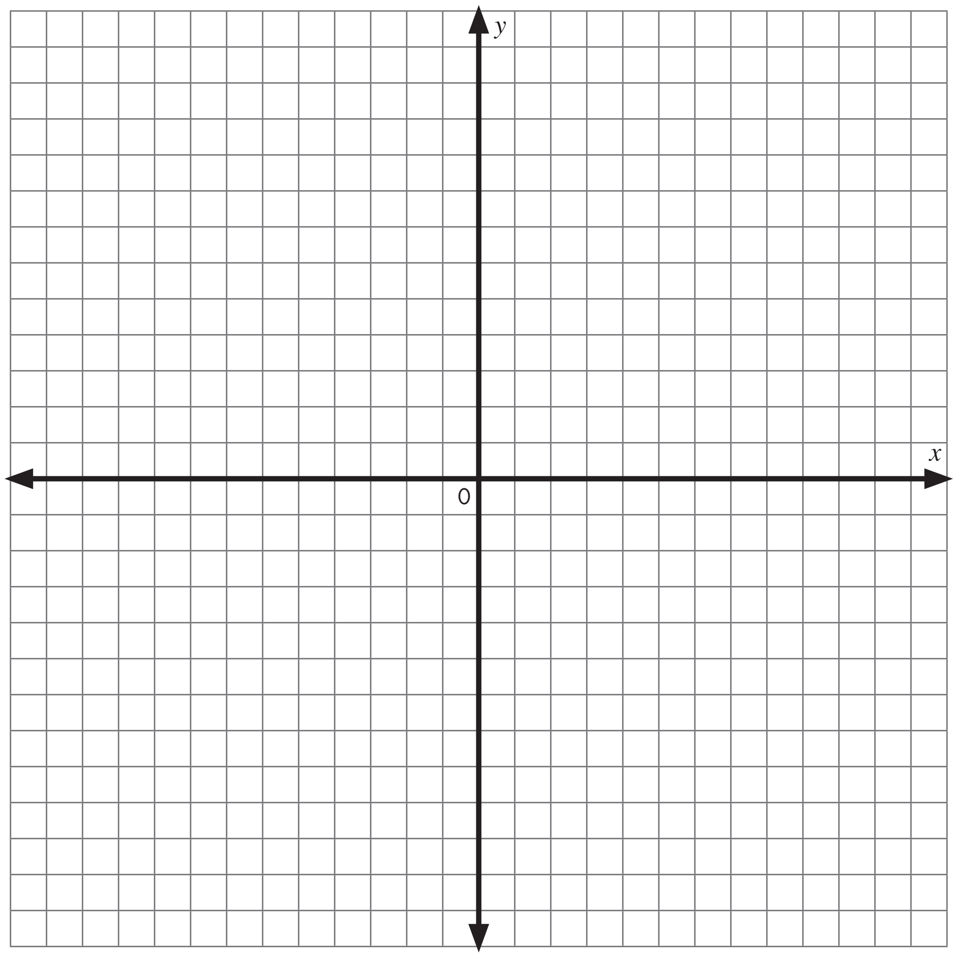
2. The inequality $x + 4 \geq y$ would have a ____________________ line.
(choose broken or solid)

3. Graph the following inequalities.

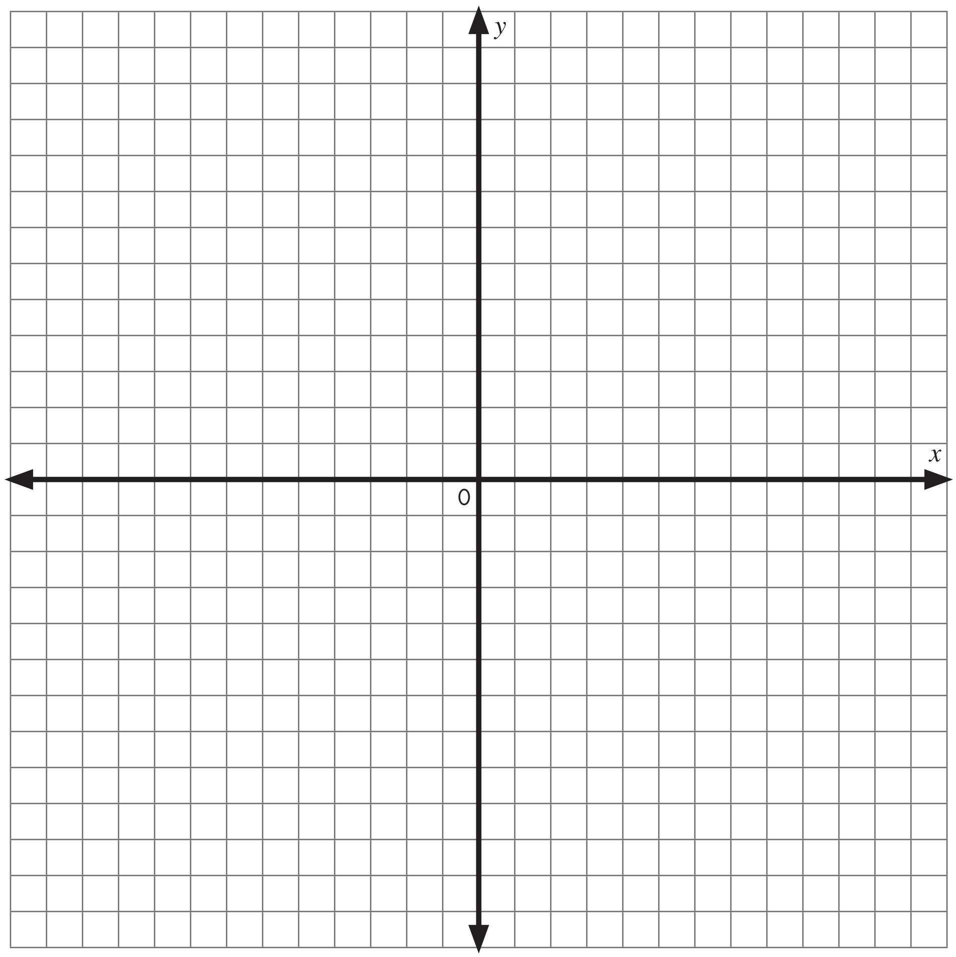
a. $2x + 8 > y$



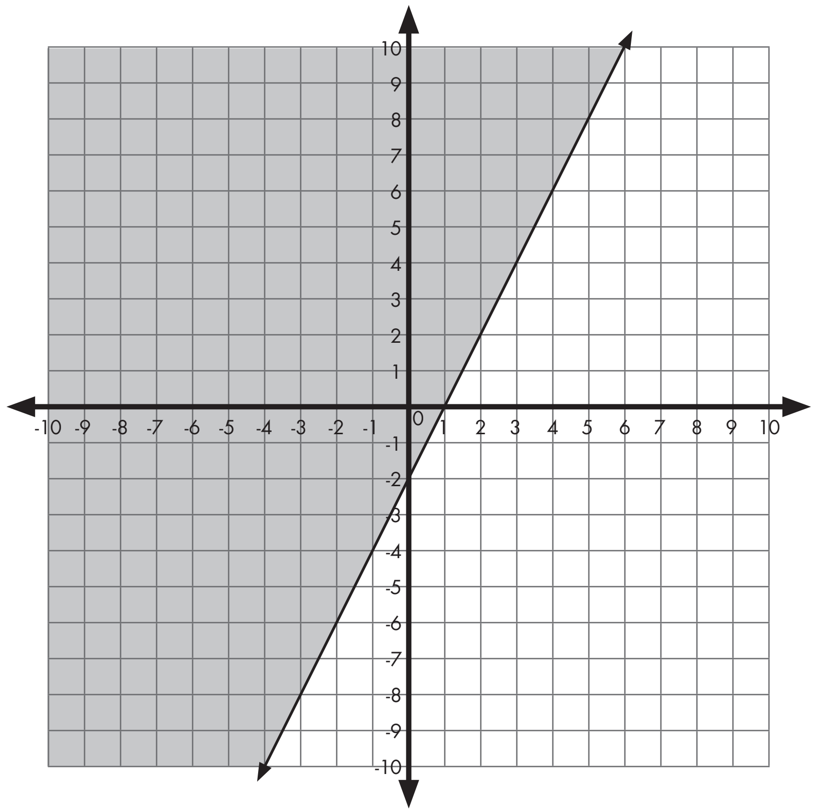
b. $y \geq -x + 1$



c. $4x + 3 > y$



4. What inequality is graphed below?



5. Identify three solutions to the inequality above.

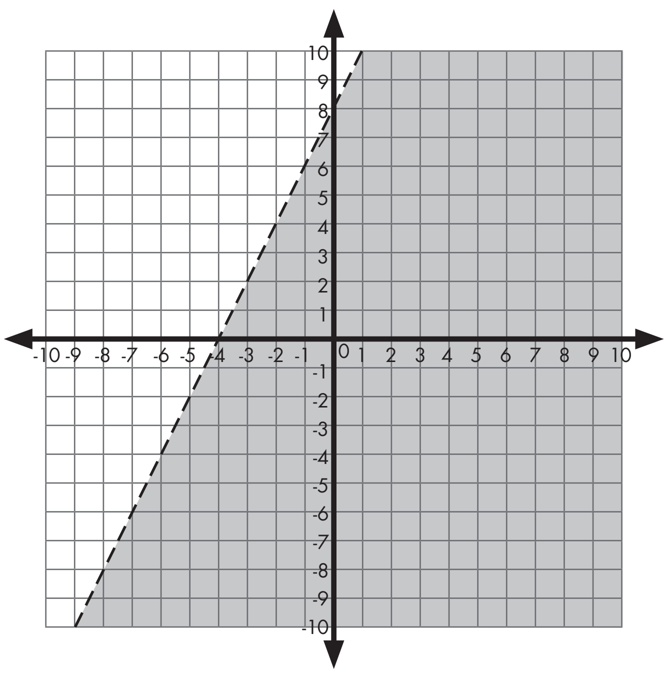
Answer Key: Graphing Inequalities
Summative Evaluation

1. The inequality $y < 2x - 2$ would have a <u>broken</u> line.

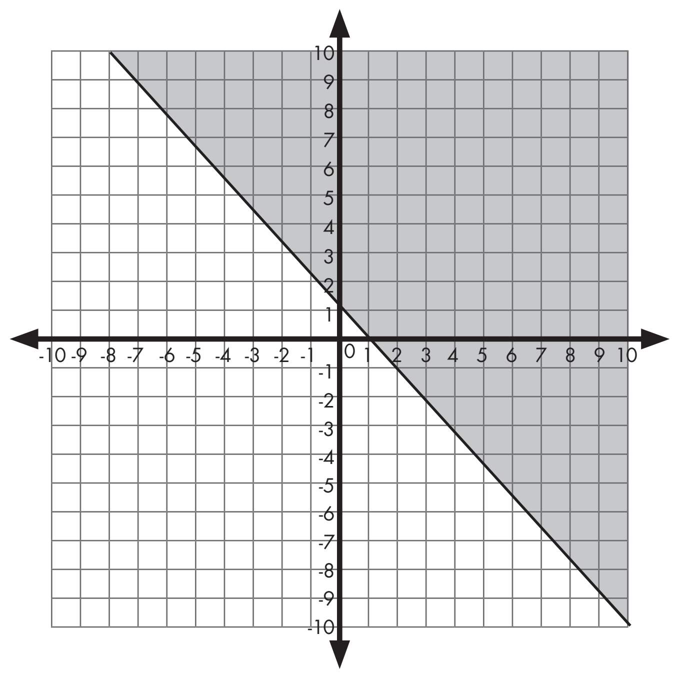
2. The inequality $x + 4 \geq y$ would have a <u>solid</u> line.

3. Graph the following inequalities.

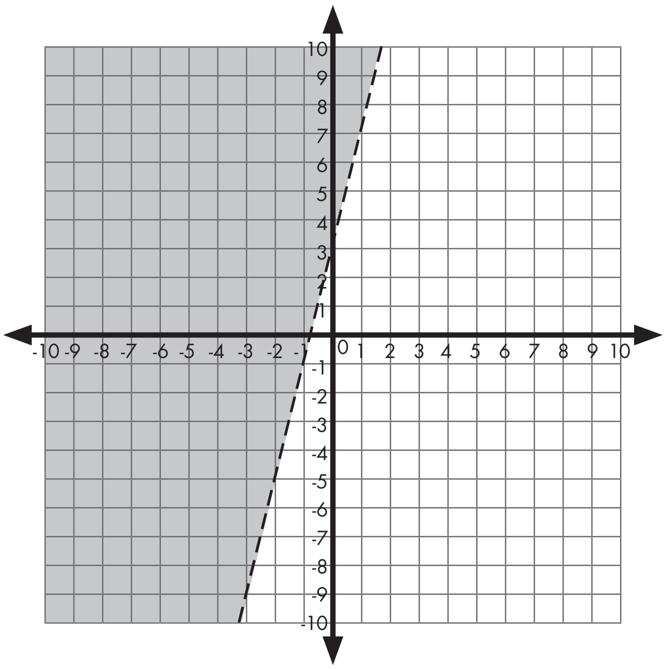
a. $2x + 8 > y$



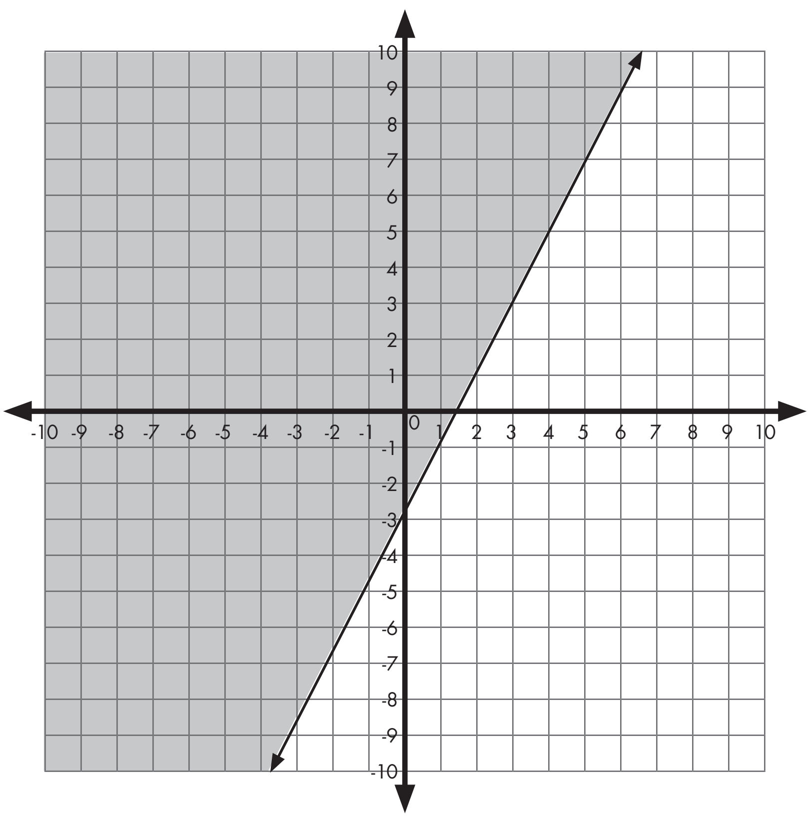
b. $y \geq -x + 1$



c. $4x + 3 > y$



4. What inequality is graphed below? $\underline{\quad y \geq 2x - 3 \quad}$



5. Identify three solutions to the inequality above.

Answers will vary. Possibilities include (-6, 0) (2, 6) (-8, -2)

Algebra Vocabulary

Binomial: A polynomial of only two terms.

Coefficient: The numeric factor in a term; e.g., the number 3 in the term *3x2y* is the coefficient. In the term *a3b*, 1 is the coefficient.

Coordinate plane: A plane in which an ordered pair can be located by reference to two intersecting number lines.

Coordinate of a point: The number paired with that point on a number line.

Coordinates of a point: See ordered pairs.

Constant: A monomial consisting of a numeral only; a term with no variable factor.

Function: A mathematical relationship between two variables, an independent variable and a dependent variable, where every value of the independent variable corresponds to exactly one value of the dependent value.

Inequality: A mathematical sentence that includes one of the inequality symbols <, >, ≤, ≥, or ≠ to compare unequal expressions.

Equation: A statement that shows two mathematical expressions that are equal to each other.

Horizontal axis: The horizontal number line in a coordinate plane, also called the x-axis.

Linear equation: An equation whose graph on a coordinate grid is a straight line.

Monomials: An algebraic expression which is a product of constants and *variables*.

Ordered pairs: A pair of numbers for which the order of the numbers is important. (2, -3) is an ordered pair.

Polynomials: The sum of monomials; e.g., *2a2 + 4a – 5*.

Quadrants: The two axes of a coordinate system divide the plane into four separate sections known as quadrants. These are identified as the first, second, third, and fourth quadrants.

Slope of a line: If (x_1, y_1) and (x_2, y_2) are two different points on a line, the slope of the line is given by

$$\frac{y_2\text{-}y_1.}{X_2\text{-}y_2}$$

A horizontal line has a slope of 0 and a vertical line has no slope.

Slope-intercept form of an equation: The equation of a line in the form $y = mx + b$, where m is the slope and b is the y-intercept.

Solution of a sentence: Any value of a variable that turns an open sentence into a true statement.

Solve an equation: To find the set of all solutions of the equation.

Standard form of a linear equation: $ax + by = c$, where *a, b* and c are integers and *a* and *b* are not both zero.

Univariate data: Having one variable.

Variable: A changing quantity, usually a letter in an algebraic equation or expression, that might have one of a range of possible values.

Vertical axis: The vertical number line in a coordinate plane.

X-axis: The horizontal axis in the coordinate plane.

X-intercept: The x-coordinate of a point where a graph intersects the x-axis.

Y-axis: The vertical axis or a coordinate plane

Y-intercept: The y-coordinate of a point where a graph crosses the *y*-axis.

math nMotion

Using Data to Make Decisions

WESTERNRESERVE
PUBLIC MEDIA

Chapter Synopsis

We are inundated with information — about our health, our leisure activities, sports and so much more. Information is just information. It is our responsibility to process that information and to use it wisely. That is one of the reasons we study data analysis. We need to not only evaluate this information, but also to base personal decisions on evidence (data). We also need to see the dangers of acting on decisions that are not supported by the evidence. The question then becomes, "How do we get that evidence?" Of course, the answer is by analyzing the data that we are using to make those decisions. This module shows us how race car drivers use data to help them win races.

Lesson one has students look at data about five types of cars (sedans, SUVs, sports cars, vans and hybrids) and construct graphs displaying the data. They then write about what their graph tells them and if they show any association between the categories they selected

The second lesson lets students be part of a road rally. They set a Hot Wheels car at different locations on a ramp and measure the distance that it travels. They then graph their results. Using their data, they enter the car rally. The winner is the car that gets closest to the edge of the table without going over the edge. (This distance is determined by graphing the data they gathered. For enrichment, students can either interpolate or extrapolate the correct distance.).

Summative and formative assessments are included, in addition to a resource page on making and understanding scatter plots and a vocabulary page.

Standards Addressed

Grade 8, Mathematics — Data Analysis and Probability

08-10 Benchmark **A.** ***Create, interpret and use graphical displays and statistical measures to describe data; e.g., box-and-whisker plots, histograms, scatterplots, measures of center and variability.***

Y2003.CMA.S05.G08-10.BA.L08.I01 / Data Collection

01. Use, create and interpret scatterplots and other types of graphs as appropriate.

Scatter Plots

You would use line plots for the following reasons:

- They organize data using bivariate data (two variables).
- They are similar to coordinate plane graphing.
- They show association between the variables being graphed:
    - Positive association – points show a trend of moving up and to the right
    - Negative association – points show a trend of moving down and to the right
    - No association – points show no trend.
- There is a need to stress the difference between association and causation
- They show clusters and outliers.
- You can use time on either axis, but it is generally the independent variable and is on the x-axis.
- In time series plots, you may connect points.

You would use lines on a scatter plot for the following reasons:

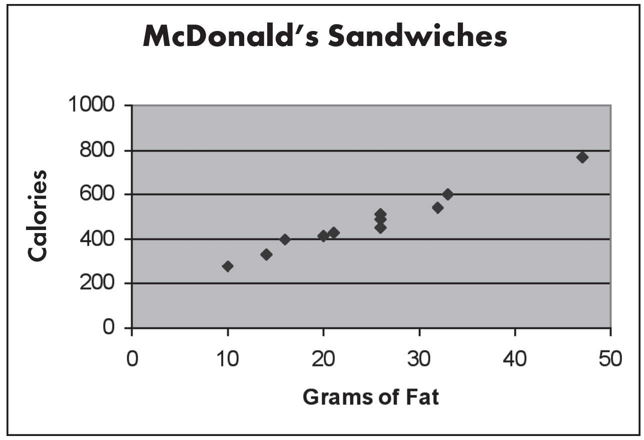
- The line divides the plot into two regions by using a 45-degree line called the $y = x$ line.
- The line distinguishes characteristics between points on, above, or below the line.
- The line can be used to make predictions. This fits in particularly well in an algebra course in a unit on equations of lines, slopes, intercepts, etc.

To make a median fit line, you would:

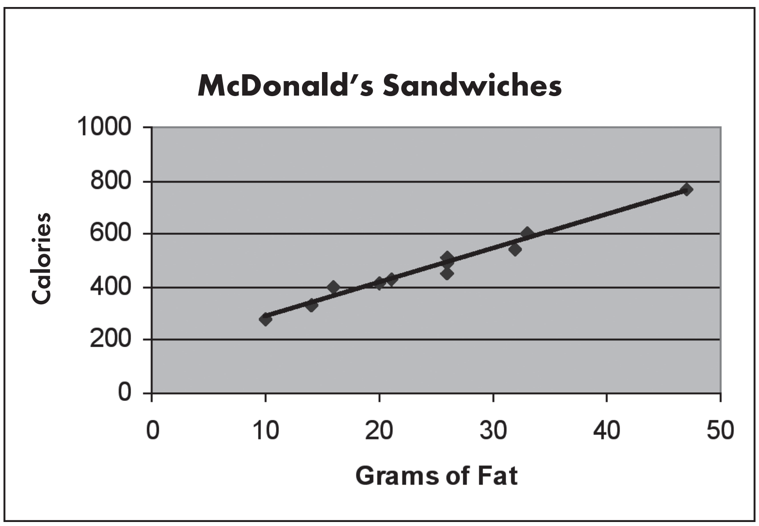
1. Count the total number of points.
2. Draw two vertical dashed lines so there are approximately the same number of points in each of the three sections. The outer strips should have the same number of points if possible.
3. Find the median point both vertically and horizontally and put a vertical line and a horizontal line at that point, forming an X.
4. Decide whether or not the three X's lie close to a straight line. Use your ruler to help you decide.
5. Place your ruler so that it connects the two X's in the outside strips. Now slide the ruler one-third of the way to the middle X and draw the line.

Information About Sandwiches at McDonalds Restaurant

Sandwich	Fat grams	Calories
Hamburger	10	280
Cheeseburger	14	330
Double Cheeseburger	26	490
Chicken McGrill	16	400
Filet-o-Fish	20	410
Quarter Pounder	21	430
Quarter Pounder/cheese	47	770
Hot N' Spicy McChicken	26	450
Crispy Chicken	26	510
Big N' Tasty	32	540
Big Mac	33	600



Scatter Plot With Linear Trend Line



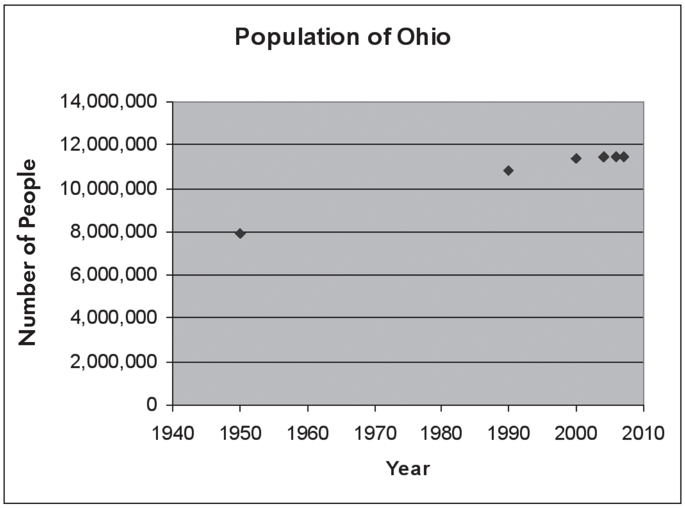
To make a median fit line on your graphing calculator

- Go to **Stat**
- Select **Calc**
- Select **Med-med** or **#3**
- **Second List1** (using the list you want to use)
- **Comma**
- **Second List2** (using the list you want to use)
- **Enter** (you'll get the line information on the screen)
- Go to **Y=** (select the "y" you want to use)
- Go to **Vars**
- Select **Statistics** or **#5**
- Arrow over to **EQ**
- **RegEQ** will be highlighted so **Enter**
- Then hit **Graph** (make sure your window has been set)

Name ______________________________

Formative Assessment

1. Below is a graph that gives the population statistics for Ohio from 1950-2007. Write two sentences that explain the graph.



2. Below is a table showing the height of a person and the length of his or her foot. Make a scatter plot of this data.

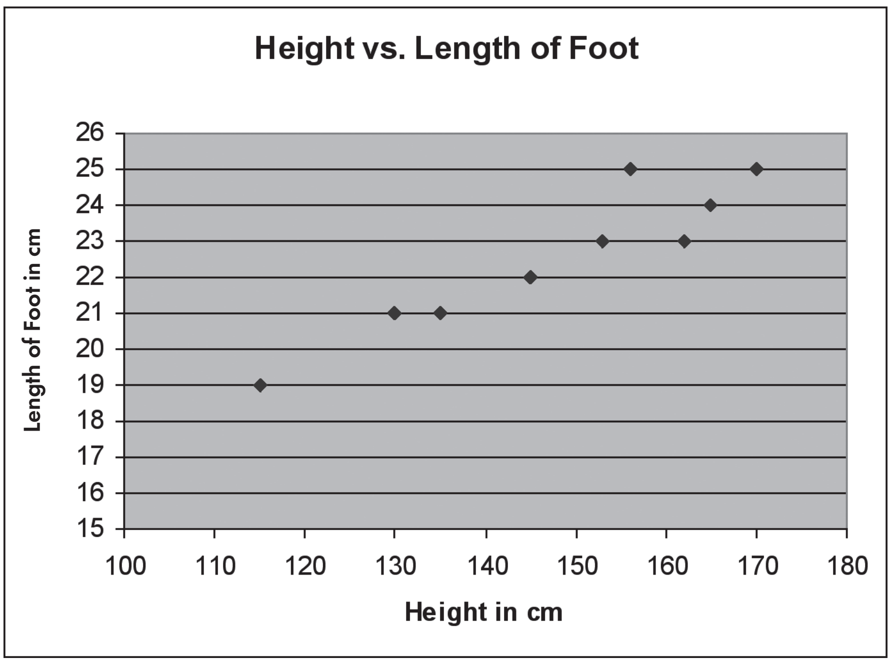
Height (cm)	Length of Foot (cm)
115	19
130	21
135	21
145	22
153	23
156	25
162	23
165	24
170	25

c. What is the independent variable? ______________________________

d. What is the dependent variable? ______________________________

e. Why?

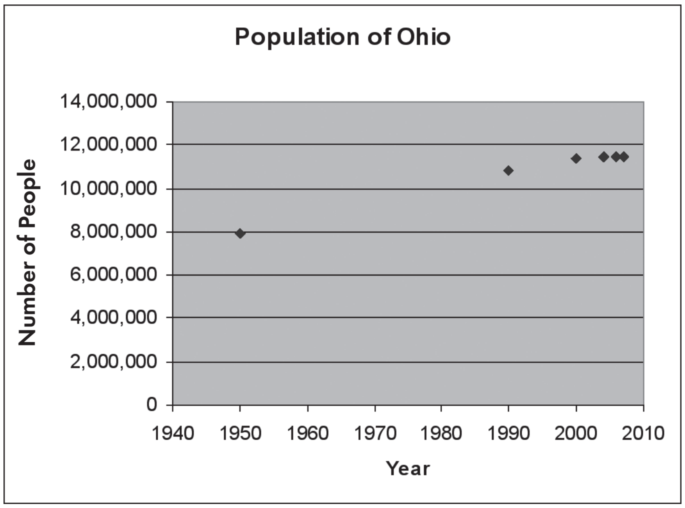
f. Estimate the length of the person's foot if he or she person is 100 cm tall. ______________



Write two sentences that tell about this graph.

Formative Assessment — Answers

1. Below is a graph that gives the population statistics for Ohio from 1950-2007. Write two sentences that explain the graph.



Possible answers: The graph shows the population of Ohio from 1950 to about 2007. The year is the on the x-axis and the population is on the y-axis. It looks as if there was an increase in population from1950 to about 2000. Also, it appears that the population has remained stable from about 2000 on.

2. Below is a table showing the height of a person and the length of his or her foot. Make a scatter plot of this data.

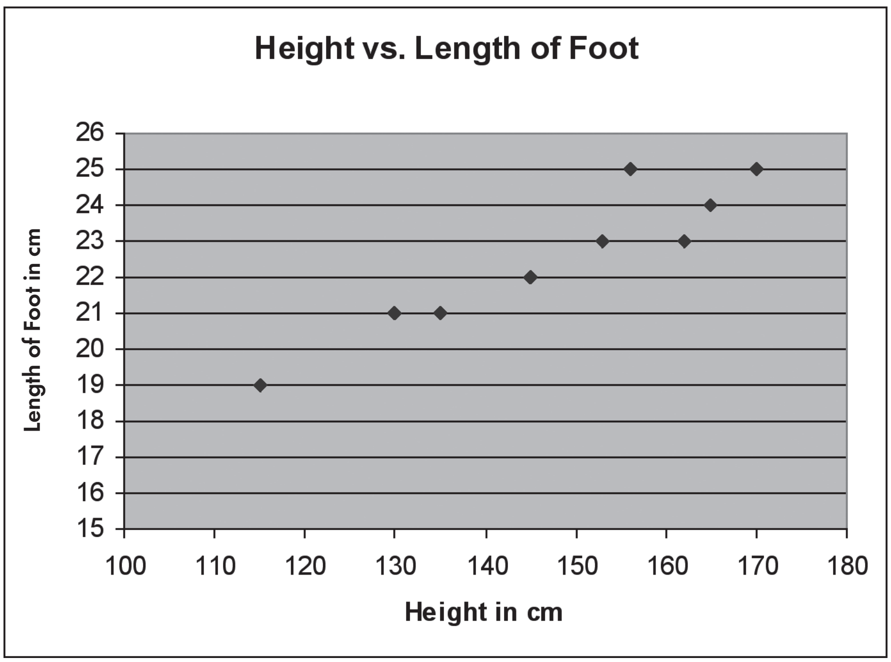
Height (cm)	Length of Foot (cm)
115	19
130	21
135	21
145	22
153	23
156	25
162	23
165	24
170	25

c. What is the independent variable? height

d. What is the dependent variable? length of foot

e. Why? The length of the foot is generally dependent upon the height.

f. Estimate the length of the person's foot if he or she person is 100 cm tall. The person's foot would probably be 18-19 cm long.



Write two sentences that tell about this graph.

Possible answers: Generally, the taller a person is, the bigger his or her foot is. There is a positive association. The x-axis is the height and the y-axis is the length of the foot.

Graphing Data

Overview

Students will look at data about five types of cars and construct graphs displaying the data. They will then write about what their graph says and if they show any association between the categories.

Standards Addressed

Grade 8, Mathematics — Data Analysis and Probability

08-10 Benchmark ***A.*** ***Create, interpret and use graphical displays and statistical measures to describe data; e.g., box-and-whisker plots, histograms, scatterplots, measures of center and variability.***

Y2003.CMA.S05.G08-10.BA.L08.I01 / Data Collection

01. Use, create and interpret scatterplots and other types of graphs as appropriate.

Materials

- Graph paper
- Computers (optional)

Procedure

1. Ask the students to name different types of cars, such as SUVs.

2. Distribute student handout Types of Cars to students either individually or in pairs. Review the column headings on the table. Explain any that they do not understand or have someone in the class explain it.

3. Optional (for younger students): Give the students one or more sticky notes each and ask them to write down the type of car their family owns. Make a line plot on the board showing the classroom data.

4. Tell the students that they are to create two graphs. Only one can be a bar graph. They are to make at least one scatter plot that compares two of the category headings. You can decide if the students may use a computer to create the graph or if they are to do it by hand using Excel or Create-a-Graph. (There are many types of graphs that could be made using this data. Some samples are given at the end of this lesson.)

5. When they have completed their graphs, ask the students to write at least two sentences that explain each graph. They are to tell if they see a relationship between any two categories. Some observations they might make include the following:

    - The higher the cost, the more horsepower
    - Highway driving allows you get higher mileage than city driving

- There is no correlation between the size of the tank and miles per gallon

6. Instruct the students to answer the questions at the bottom of the student handout.

7. Ask for volunteers to share their graphs with the class. Their work could be taped to the board or the wall or shown on a document camera. Continue having the students share their work until the graphs start to be duplicated.

Answers to student handout

1. Yes, they are representative of the categories. Many other cars could have been chosen.

2. It is really not a sufficient sample. More cars from each category should have been used.

3. The mean, median and mode would not be useful in this context. (It's similar to finding the means of the jerseys of a football team. You can do it, but the mean or median are not useful data.) If you had more samples, the mode might be useful; however, with this data, it is not useful either. The range might be a useful number because it shows the spread of the data in a variety of categories.

4. Answers will vary.

Evaluation

Rubric for Graphs

CATEGORY	4	3	2	1
Title	The title is creative and clearly relates to the problem being graphed. It is printed at the top of the graph.	The title clearly relates to the problem being graphed and is printed at the top of the graph.	A title is present at the top of the graph.	A title is not present.
Labeling of x-axis	The x-axis has a clear, neat label that describes the units used.	The x-axis has a clear label.	The x-axis has a label but it is unclear.	The x-axis is not labeled.
Labeling of y- axis	The y-axis has a clear, neat label that describes the units.	The y-axis has a clear label.	The y-axis has a label but it is unclear.	The y-axis is not labeled.
Accuracy of Plot	All points are plotted correctly and are easy to see. A ruler is used to neatly connect the points or make the bars, if not using a computerized graphing program.	All points are plotted correctly and are easy to see.	All points are plotted correctly.	Points are not plotted correctly, or extra points were included.
Units	All units are described (in a key or with labels) and are appropriately sized for the data set.	Most units are described (in a key or with labels) and are appropriately sized for the data set.	All units are described (in a key or with labels) but are not appropriately sized for the data set.	Units are neither described nor appropriately sized for the data set.
Neatness and Attractiveness	Exceptionally well designed, neat and attractive. Colors that go well together are used to make the graph more readable. A ruler and graph paper (or graphing computer program) are used.	Neat and relatively attractive. A ruler and graph paper (or graphing computer program) are used to make the graph more readable.	Lines are neatly drawn but the graph appears quite plain.	Appears messy and produced in a hurry. Lines are visibly crooked.
Concepts	Student has a clear understanding of plots and has answered the questions effectively.	Student has satisfactory understanding of the major concepts, but has small misunderstandings.	Student has major misunderstandings of the concepts and cannot complete work on his own.	Student does not display understanding of the major concepts or did not complete the assignment.

Name ______________________________

Types of Cars

Make and Model	MPG — City	MPG — Hwy	Basic Cost in $	Horsepower	Cargo Volume	Fuel Tank
Honda Accord LX (sedan)	22	31	20,905	177	14 cu ft	18.5 gal
Hummer H3 (SUV)	14	18	34,135	239	25 cu ft; 62.8 cu ft with seat down	23 gal
Chevrolet Corvette (sportscar)	16	26	49,415	430	22.4 cfeet	20 gal
GM Savana Passenger 1500 (van)	13	16	29,455	301	204 cu ft with seats; 217.3 cu ft without seats	31 gal
Honda Civic (hybrid)	40	45	23,650	110	10.4 cu ft	12.3 gal

Above is data from five different types of cars: a sedan, an SUV, a sports car, a van and a hybrid. Your job is to make two graphs. Only one can be a bar graph. Then write two sentences to describe each of your graphs. If you see that one category has an effect on another category, please include that in your definition. You may create your graph by hand, use Excel or use Create-a-Graph.

1. Do you believe that these cars are representative of other cars in their category? Why or why not.

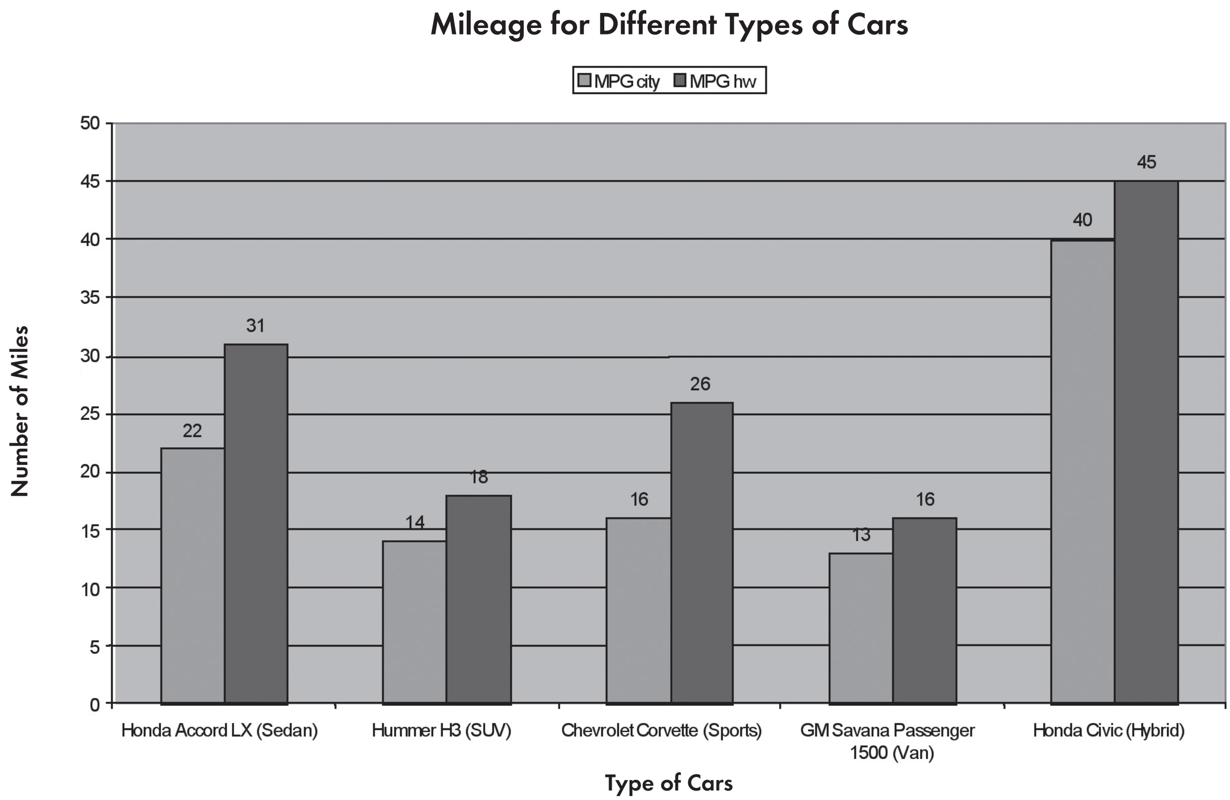
2. Is this a sufficient sample for each type of car? Why or why not?

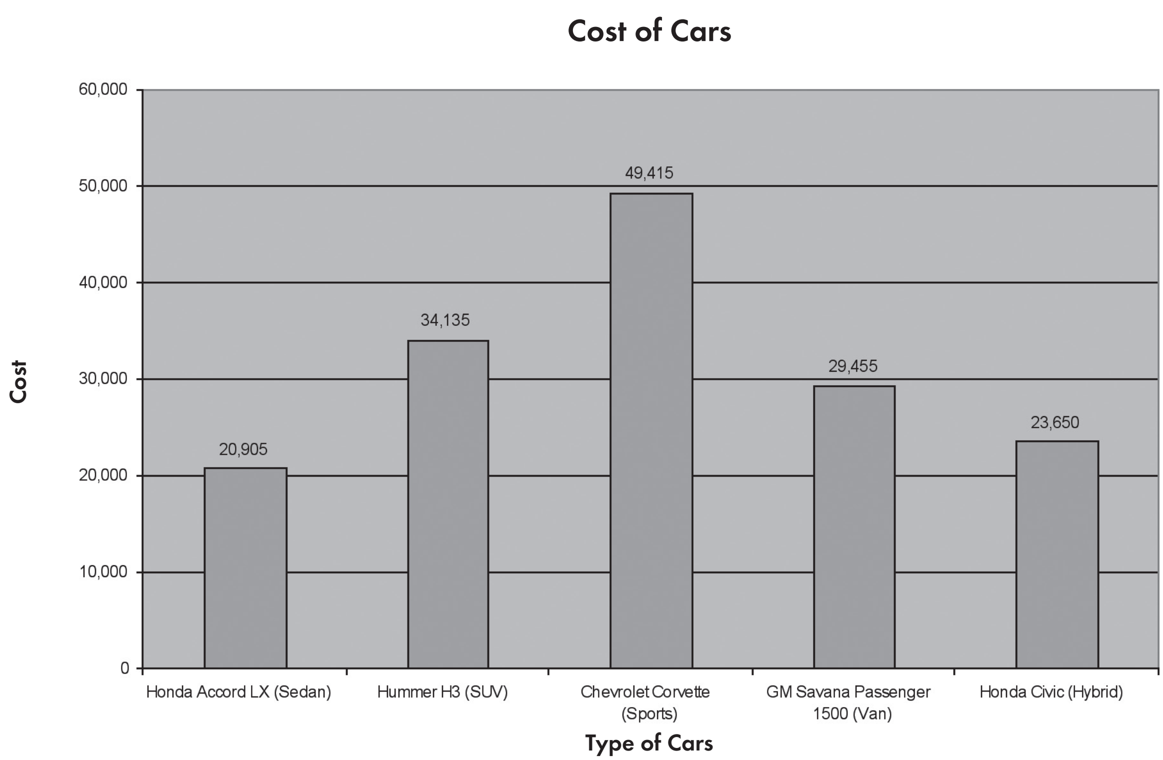
3. Should we find the mean of any category? The median? The mode? The range? Why or why not?

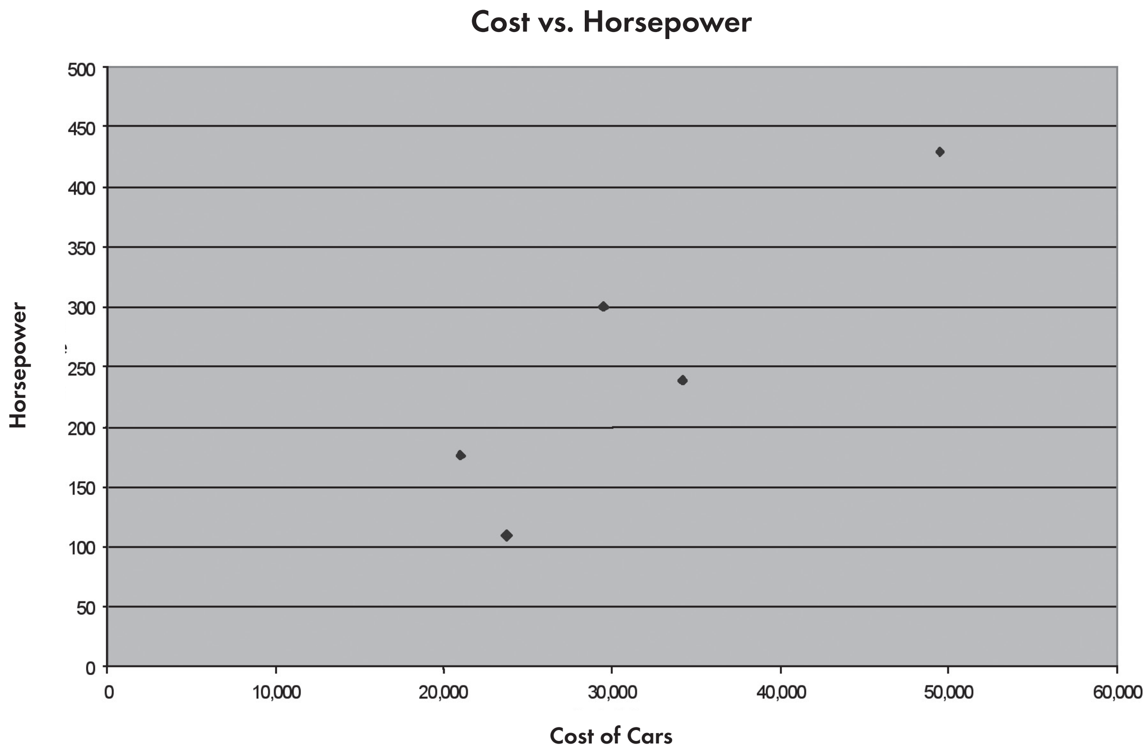
4. Add a category to the graphs using the Formula M racer data. Does the addition of this data change your mind about anything that you wrote above?

Make and Model	MPG	Basic Cost in $	Horsepower	Cargo Volume	Fuel Tank
Formula M Racer	10	40,000	180	0	7 gal

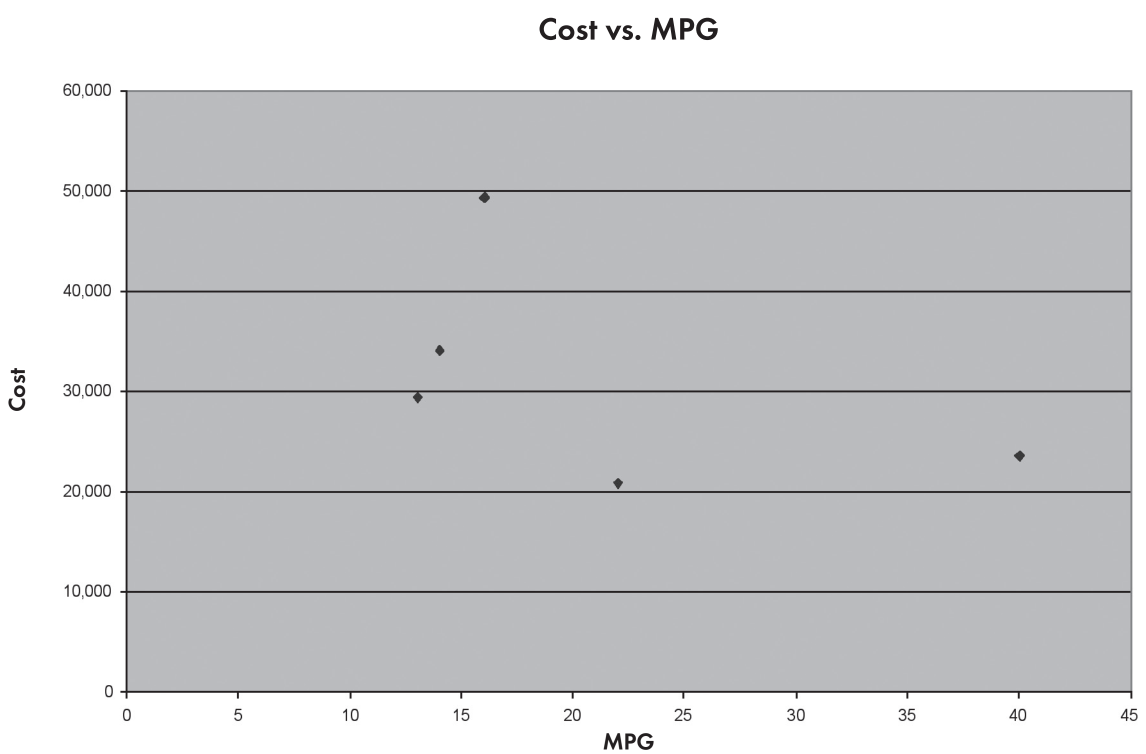
Sample Graphs







The above graph shows a positive association. The higher the price, the greater the horsepower.



This graph shows no association between miles per gallon and cost.

Road Rally Graphing

Overview

Students will put a Hot Wheels car at different locations on a ramp and measure the distance that it went. They will then graph their results. Using their data, they will enter the car rally. The winner is the car that gets closest to the edge of the table without going over the edge. (This distance will be determined by graphing the data they gathered. For enrichment, students can either interpolate or extrapolate the correct distance.)

Standards Addressed

Grade 8, Mathematics — Data Analysis and Probability

08-10 Benchmark **A.** ***Create, interpret and use graphical displays and statistical measures to describe data; e.g., box-and-whisker plots, histograms, scatterplots, measures of center and variability.***

Y2003.CMA.S05.G08-10.BA.L08.I01 / Data Collection

01. Use, create and interpret scatterplots and other types of graphs as appropriate.

Materials

- Cardboard for ramps
- Masking or duct tape
- Hot Wheels cars
- Tape measures
- Graph paper
- Rulers
- Reward items

Procedure

1. Divide the class into pairs and tell the students that they are going to be part of a road rally team.

2. The students can either make a ramp or you can provide ramps. The ramp should be about 16 inches long. Have the students draw lines on the ramp about 2-4 inches apart. Label the distances from the top of the ramp.

2"							

3. Put a table next to a wall for each ramp, with the short side of the table abutting the wall.

4. Instruct the students to affix the marked ramp with the high end against the wall. They can use any angle of measure of the ramp to the wall, but 45 degrees works well. They may need to put a piece of tape at the bottom of the ramp so the cars can roll smoothly onto the table.

5. Have the students name their cars. (Even though the students are working with a partner, it is good for each student to have a car.)

6. Tell them that in this road rally, the car that comes the closest to the edge of the table without going over is the winner. (To find the distance to the edge, measure the distance from the edge of the table to the front bumper of the car.) The students place the car at one of the marked lines on the ramp and let it go. They then measure the distance that the car travelled. Each car can have three trial runs before the event.

7. Students are to make a graph of their results. On the x-axis, they are to indicate at what inch mark on the ramp they started, measuring from the top. This is the independent variable. The y-axis should show the distance that they travelled (dependent variable).

8. They will know the length of the table. They should be able to put that length onto the graph and determine how many inches down the ramp they should put their car to get it close to the edge of the table.

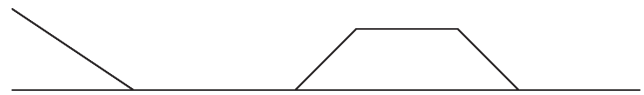
9. Reinforce the idea of collecting data to make an informed decision.

10. Begin the rally, where the teams use either one or both of their cars to determine the winner. Give each team two or three tries to get to the edge.

11. Offer a reward to the winner(s).

Enrichment 1: Get a longer table or a shorter table and ask the students to use their graph to either interpolate or extrapolate the distance down the ramp they would go to be close to the edge of the table. Run the contest again.

Enrichment 2: Have a ramp, a table and a hill as the course.



The goal is to have the car stop on top of the hill. Students should follow the same procedure in testing their cars and recording the results. A second contest would then take place. All who get their cars to stop on top of the hill would be winners. There could be more than one try to achieve the goal.

Enrichment 3: The rally could be done by varying the height of the ramp or the weight of the car.

Evaluation

_______ Driver and crew names are written (3).

_______ Measurements of the car are listed (7).

_______ Trial distances are listed on chart (10).

_______ Trials are graphed correctly (20).

_______ Car came close to the edge during the contest (10).

Total Points (50)

Road Rally

Driver: ____

Crew: ____

Name of car: ____

Car stats: Length: ____________________ Width: ____________________

Trial #	Start: Distance From Top of Ramp	End: Distance From Edge of Table
Trial 1		
Trial 2		
Trial 3		
Trial 4		

Rally Report

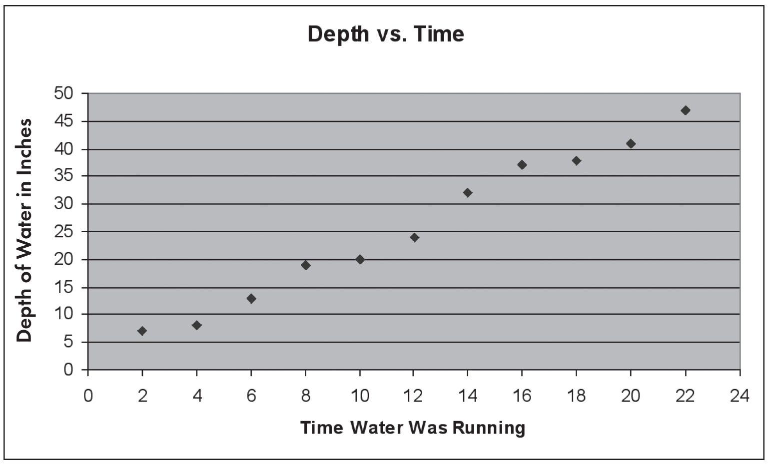
Distance from the edge of the table in ______

Distance from the top of the ramp in ______

Name ____

Summative Assessment

1. For a science project, a student measured the amount of time that water was running in a bathtub and the corresponding depth of the water. Write two sentences that explain this graph.



Data from http://www.icoachmath.com/SiteMap/ScatterPlot.html

2. Steve wondered if studying really made a difference in test scores so he asked his classmates how long they studied and what their test score was. Make a scatter plot of the data that he collected.

Study Hours	**Exam Score**
3	80
4	90
2	70
6	80
7	85
1	50
2	65
7	80
1	40

a. What is the independent variable? ____________________

b. What is the dependent variable? _______________________

c. Why?

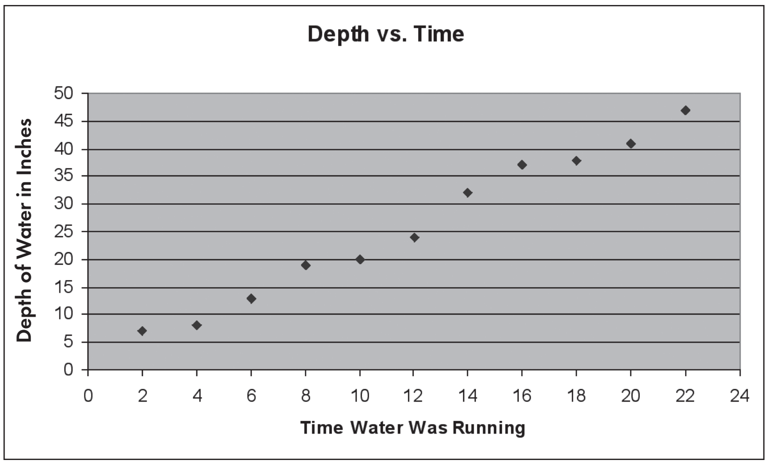
d. Estimate the score if you studied for five hours.

Write two sentences that tell about this graph.

Name ____________________

Summative Assessment — Answers

1. For a science project, a student measured the amount of time that water was running in a bathtub and the corresponding depth of the water. Write two sentences that explain this graph.



Data from http://www.icoachmath.com/SiteMap/ScatterPlot.html

Possible answers: The graph shows the relationship between the time spent filling the tub with water and the depth of the water. Generally the more time spent filling the tub with water, the greater the depth. The x-axis is the independent variable because the depth depends on the amount of time. Depth would be the dependent variable.

2. Steve wondered if studying really made a difference in test scores so he asked his classmates how long they studied and what their test score was. Make a scatter plot of the data that he collected.

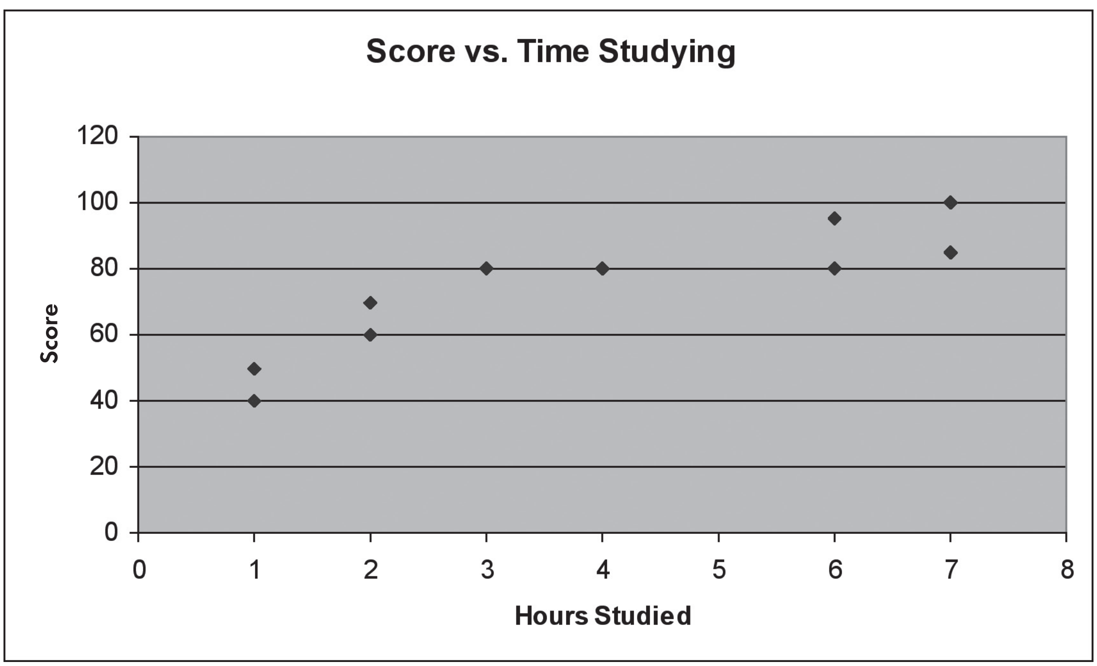
Study Hours	**Exam Score**
3	80
4	90
2	70
6	80
7	85
1	50
2	65
7	80
1	40

a. What is the independent variable? hours studying

b. What is the dependent variable? exam score

c. Why? Generally, the more hours studying, the higher the score.

d. Estimate the score if you studied for five hours. 80 to 85



Write two sentences that tell about this graph.

Possible Answers: The general trend is that the more hours you study the better your score. The graph shows a slight positive association. The hours studied is the independent variable because the score is dependent upon the hours.

Graphing Vocabulary

Association: The trend in a graph that shows the effect on one measure by a second measure.

Bivariate data: Data or events described using two variables.

Box-and-whisker plot: A diagram that shows pictorially the median and measures of spread (upper and lower interquartile ranges and the range) for one set of data. Example:

Data = 35, 25, 90, 60, 45, 40, 58, 90, 90, 55, 60, 55, 80, 90, 60, 55, 60, 85, 75, 60, 56, 55, 75, 80, 90

Minimum = 25

Median = 60

Maximum = 90

Lower quartile = 55

Upper quartile = 83

Broken-scale symbol (/\/\/\): Symbol is Is used at the end of the axis to indicate that the graph does not begin at the origin (0,0).

Causation: The relationship between two variables where a change in one variable affects the outcome of the other variable.

Categorical data: Data that can be classified by type; e.g., color, types of dogs. These types of data are typically represented using bar chart, pie charts or pictographs.

Clusters and gaps: Numbers that tend to crowd around a particular point in a set of values. The spaces between the clusters are called gaps.

Coordinate plane: A plane determined by the intersection of two perpendicular number lines in which any point can be located.

Coordinates: An ordered pair of numbers used to show a position on a graph. The first number (x) gives the place left or right and the second number (y) gives the place up or down (x,y).

Correlation: The relation between two sets of data, a positive or direct correlation exists when both sets vary in the same direction (both sets decrease); a negative or inverse correlation exists when one set of data increases as the other decreases.

Datum: A single piece of data. The singular form of data.

Dependent events: A statement or probability for one event affects a statement or probability for another event.

Descriptive statistics: Gathered and described data using probability, statistical methods and concepts like graphs and measures of center.

Dispersion: How data is spread out around some central point.

Distribution: A graph or table showing how many pieces of data there are in each class, or of each type.

Equation: A statement that shows two mathematical expressions that are equal to each other.

Experiment: The collecting of data through a planned investigation.

Extrapolation: A term used in interpreting scatter plots that predicts the location of points extending beyond the data displayed.

Frequency: The number of times an occurrence takes place as expressed in a count. The count is the frequency.

Frequency distribution: A collection of data that represents the number of times a set of numbers, items or events has occurred.

Frequency table: A table that shows how often each item, number or range of numbers occurs in a set of data.

Histogram: Shows how measurements spread out across a real number line, marked off in equal intervals. The height of the bar represents the frequencies of measurements contained in that interval.

Intercepts: The value of y on the coordinate plane where x = 0, called the y-intercept. The value of x on the coordinate plane where y = 0, called the x-intercept.

Interpolation: A term used in interpreting scatter plots that predicts the location of points that would lie between those points of the data already displayed.

Interval: The distance on a real number scale between two consecutive tick marks or the space between the two points.

Line of best fit: A line drawn in the midst of the points on a scatter plot in an attempt to estimate the mathematical relationship between the variables used to generate the plot.

Linear equation: An equation whose graph on a coordinate grid is a straight line.

Measurement data (quantitative): Has a numerical value and could be placed on a number line.

Measures of center: Numbers that provide information about cluster and average of a collection of data. They include the mean, mode and median.

Measures of spread or variability: A term used to refer to how much numbers are spread, varied or dispersed in a set of data. They include range, quartile and interquartile range.

Mean: The sum of a set of numbers divided by the number of elements in the set.

Median: The middle number or item in a set of numbers or objects arranged from least to greatest, or the mean of the two middle numbers when the set has two middle numbers.

Mode: The number or object that appears most frequently in a set of numbers or objects.

Negative association: A pattern in the shape of the data that shows when one measurement grows larger, the second measure grows smaller. A negative association will be slanted downward from left to right.

Ordered pairs: A pair of numbers that gives the coordinates of a point on a grid in this order (horizontal coordinate, vertical coordinate). Also called paired data or paired coordinates.

Origin: The point on a coordinate plane where the x-axis and the y-axis meet and have the ordered pair (0,0).

Outlier: A data point in a sample widely separated from the main cluster of points in the sample.

Parallel box plots: Two or more box plots using the same number line to allow comparison of the data.

Positive association: The pattern in the shape of data that shows when one number gets larger, the second also gets larger. It will be slanted upward from the left to right.

Quadrants: The four separate sections formed by the two axes of a coordinate system. These are identified as the first, second, third and fourth quadrants.

Quartile: In conjunction with the median, the quartiles divide the set of data into four groups of equal size.

Range: The difference between the greatest and the least numbers in a set of data.

Scale: The regular intervals of the number line that are chosen to represent the full range of data on a graph.

Scatter plot: A graph with one point for each item being measured. The coordinates of a point represent the measure of two attributes of each item.

Slope of 1: A line that contains points where the abscissa (x value) and the ordinate (Y value) are equal. (The rise equals the run.)

Stem-and-leaf plot: A frequency diagram that displays the actual data together with its frequency, by using a part of the value of each piece of data to fix the class or group (the stem), while the remainder of the value is actually listed (the leaves).

Symbolic form: A representation of something using numbers and symbols.

Trend: An emerging pattern in the shape of a data display that can be seen on a scatter plot.

Univariate data: Having one variable.

Upper extreme: The largest value in a set of data; the maximum.

Upper quartile: When data is ordered from smallest to largest and divided into four quarters, the values that are in the upper quarter of the data.

Variable: A changing quantity, usually a letter in an algebraic equation or expression, that might have one of a range of possible values.

X-axis: The horizontal axis on a coordinate grid.

Y-axis: The vertical axis on a coordinate grid.

Definitions are from the Ohio State Content Standards for Mathematics and Exploring Statistics in the Elementary Grades by Bereska, Boster, Bolster and Scheaffer. (Dale Seymour Publications)

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